Okun's Law Revisited in the Time-Frequency Domain: Introducing Unemployment into a Wavelet-Based Control Model

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Abstract. This paper integrates Okun's Law relationship into a wavelet-based control (WBC) model to compare simulated optimal fiscal and monetary policy when the policymakers place varying emphasis on the primary macroeconomic targets of unemployment, output growth, and inflation. The simulation results show that the unemployment rate is impacted differently across frequency ranges. We find that fiscal policy is the most aggressive when economic growth is emphasized as a policy objective whereas monetary policy is relatively more aggressive when the inflation rate is emphasized. Given that the U.S. inflation rate was below target for the start of the simulation exercises, when it is emphasized, that leads to lower interest rates, a depreciated exchange rate, and larger aggregate investment.

Keywords: Discrete Wavelet Analysis; Monetary Policy; Okun's Law; Optimal Control

JEL classifications C61 . C63 . C88 . E52 . E61 . F47

1. Introduction

In this paper monetary and fiscal policies are explored within a large-scale wavelet-based control (WBC) model of the U.S. that integrates a time-frequency expression of Okun's Law (OL). The WBC model is then used to simulate the U.S. macroeconomy under various different assumptions about both the emphasis of the policymakers and the underlying state of the economy. The main purpose of the simulations is to analyze the relative changes in the optimal forecast trajectories that occur when the fiscal and monetary policy prioritize unemployment as the base case versus the cases where the priority is on economic growth and inflation targets. Since the WBC model is not a fully calibrated large scale econometric model, the results are meant to be primarily qualitative and illustrative.

To preview the results, we find that fiscal policy is more aggressively used when economic growth is more heavily emphasized, and the main thrust of the expansion is due to the growth emphasis is at the shortest cycle length. The results also show that monetary policy is more heavily utilized when an inflation target is prioritized. In the low unemployment rate environment, the increased prioritization of economic growth and inflation both cause an increase in output growth that causes the unemployment rate to linger farther below its natural due to OL.

Time-frequency domain modeling and analysis has been used extensively in recent economics literature to provide empirical and policy insights into a number of standard macroeconomic relationships. For example, Gallegati et al. (2011) used wavelets to explore the Phillip's curve (PC), and both Aguiar-Conraria, Martins, and Soares (2018), and Crowley and Hudgins (2020) used wavelet analysis to explore the time-frequency properties of the Taylor rule (TR). More recently Aguiar-Conraria, Martins, and Soares (2020) analyzed the time-frequency aspects of Okun's law (OL) using the continuous wavelet transform (CWT) and Lubik, Matthes, and Verona (2019) and Verona (2016) use wavelets to analyze the U.S. economic cycle.

Crowley and Hudgins (2015; 2018a,b; 2019) have utilized wavelet-based control (WBC) models to simulate optimal monetary and fiscal policy. WBC models use wavelets to decompose GDP components and financial data, and then embed the decomposed relationships within a linear-quadratic optimal tracking control model that simulates jointly optimal fiscal and monetary policy. This aids aggregate forecasting by allowing policymakers to emphasize political cycle frequency ranges, and to utilize cyclical information within time series that are not captured by time domain methods.

Section 2 provides a short literature review, while section 3 considers our data and the methodology used. Section 4 presents the simulation results, while section 5 concludes.

2. Literature Review

Some recent empirical studies of aggregate versions of OL include Anderton et al. (2014), Ball, Leigh, and Loungani, P. (2017), Fernald et al. (2017), and Kamber et al. (2018). Aguiar-Conraria, Martins, and Soares (2020) provides an extensive empirical literature survey of OL, and was the first to use wavelets to analyze OL. That study estimates the OL coefficient across different episodes and different frequency ranges using CWT for the U.S. and other countries. They find that the coefficients are consistently negative for both an output-gap and a first-differences specification of OL at all frequency bands, but are time-varying, with patterns and significance that differ across cyclical frequencies. Whereas that study utilized the CWT to estimate OL, this study is the first to employ a discrete wavelet transform (DWT), specifically the Maximal Overlap Discrete Wavelet Transform (MODWT), to analyze the OL relationship.

This study focuses on simulated policy forecasts that are derived using OL as one of the state equations. Using an output-gap version of OL, Wen and Chen (2012) argue that the effectiveness of monetary policy based on OL depends upon the statistical significance of the OL coefficient, and the reliability of the specifications of the natural rate of unemployment. Using Euro-area data, Anderton et al. (2014) find that using GDP component data enhances the OL relationship, since unemployment is more responsive to consumption gaps than net export gaps, especially due to relative differences in the sector's labor productivity components.

The empirical and policy studies also consider labor productivity effects on unemployment and monetary policy. Eeckhout and Lindenlaub (2019) develops a labor market framework for a jobless recovery, and finds that the outward shift of the Beveridge curve during the boom, and for pro-cyclical frictional wage dispersion, is due to the decrease in effective job-market tightness. In that model, employed workers crowd-out the unemployed when on-the-job search picks up during economic recoveries. Hudgins and Shuai (2006) derive optimal control-based monetary policy rules when high productivity growth leads to a jobless recovery during sluggish economic growth. Aaronson et al. (2013) also employ an optimal control model that includes OL and is also based on unemployment and labor productivity to develop forecasts and analyze monetary policy.

Bova, Kolerus, and Tapsoba (2015) studies fiscal policy using OL, and finds that whereas fiscal consolidation has a positive impact on the OL coefficient, fiscal expansion does not have an impact on the OL coefficient. Pham and Sala (2019) study fiscal policy using OL, and find that fiscal shocks cause the OL coefficient to be time-varying in the short-run. The study also finds that fiscal austerity policies administered during slumps may have unexpected adverse effects on job creation if implemented during slumps.

Aguiar-Conraria, Martins, and Soares (2020) find that the OL coefficients are time-varying, with patterns and significance that differ across cyclical frequencies. However, they do not find any sensitivity of the OL coefficients to recessions, which is consistent with the aggregate OL results of Fernald et al. (2017), but inconsistent with Lee (2000).

3. Data and Methodology

Discrete wavelet analysis extracts cyclical information from time series by expressing the value of a variable x at time instant k, x_k, using Mallat's pyramid algorithm and multiresolutional analysis, as

$$x_k \approx S_{J,k} + d_{J,k} + d_{J-1,k} + ... + d_{1,k} \quad (1)$$

where the $d_{j,k}$ terms are wavelet detail "crystals", $j = 1,\ldots, J$; $S_{J,k}$ is a trend component, called the wavelet "smooth", and J represents the number of scales (frequency bands). As detailed in Crowley and Hudgins (2019), we utilize the asymmetric Daubechies 4-tap (D4) wavelet function, and employ the MODWT as the method of time-frequency decomposition. We apply the MODWT to real U.S. national income data, OECD data, U.S. inflation rate, nominal and real interest rates, and the foreign (G6=G7 minus U.S.)

real GDP weighted nominal interest rate,[1] over the post-Bretton-Woods period 1973 – 2018, using a two-step procedure that extracts the crystals and the smooth (trend and any residual cycles) at frequencies j = 1, …, 5. Table 1 defines the time-frequency ranges for all of the wavelet decompositions.

Table 1

The time intervals associated with each of the frequency ranges

J	*Time interval in quarters*	*Time interval in years*
1	2 to 4 quarters	6 months to 1 year
2	4 – 8 quarters	1 – 2 years
3	8 – 16 quarters	2 – 4 years
4	16 – 32 quarters	4 – 8 years
5	32 – 64 quarters	8 – 16 years

Based on the wavelet decomposition above, the model nests the GDP components of domestic output (Y) are in the following blocks: consumption (C_j); investment (I_j); government expenditure (G_j); net exports (NX_j). For each frequency range, each component removes the effects at all other four frequency ranges, so that a variable only includes the crystal (d) and the modified smooth base-level trend (S). The wavelet-based components for any variable are therefore defined in equation (2) as follows:

$$X_{j,k} = d_{X,j,k} + S_{X,j,k} \qquad j = 1, \ldots, 5; \quad k = 1, \ldots, K \tag{2}$$

The U.S. quarterly national income data are chain-weighted, seasonally adjusted and in 2012 prices. This data is sourced from the BEA database.[2]

Figures 1 and 2 show the wavelet decompositions for the unemployment rate and national output. In figure 1 the wavelet decomposition of the U.S. unemployment rate is shown, and it is clear that over the whole span of data considered, the largest amount of power resides in the lowest frequency crystal (d_5), which accords with frequencies between 8 and 16 years. This frequency range aligns with the longer end of the conventional business cycle. As might be expected, the second most powerful crystal (d_4) contains the other end of the conventional business cycle frequency range, aligning with 2 to 4 year cycles. There are also some interesting dynamics here, as the first part of the period d_4 was clearly dominant, but then after 1982 d_5 clearly becomes dominant. This result is consistent with the CWT work of Aguiar-Conraria, Martins, and Soares (2020), and interestingly their results also suggest that there are 2 different oscillation modes at play within the data (one at a 6 year cycle and another at a cycle between 12 and 16 years), both roughly at frequencies contained within our d_4 and d_5 crystals, with the higher frequency mode prominent in the 1970s and 80s.

[1] G6 interest rates are sourced from the OECD and US rates are sourced from the Federal Reserve. The G6 rates use real GDP in US$ weights sourced from either the IMF or OECD.

[2] See http://www.bea.gov.

Figure 1

United States Wavelet Decomposition for the U3 Unemployment Rate (***un***)

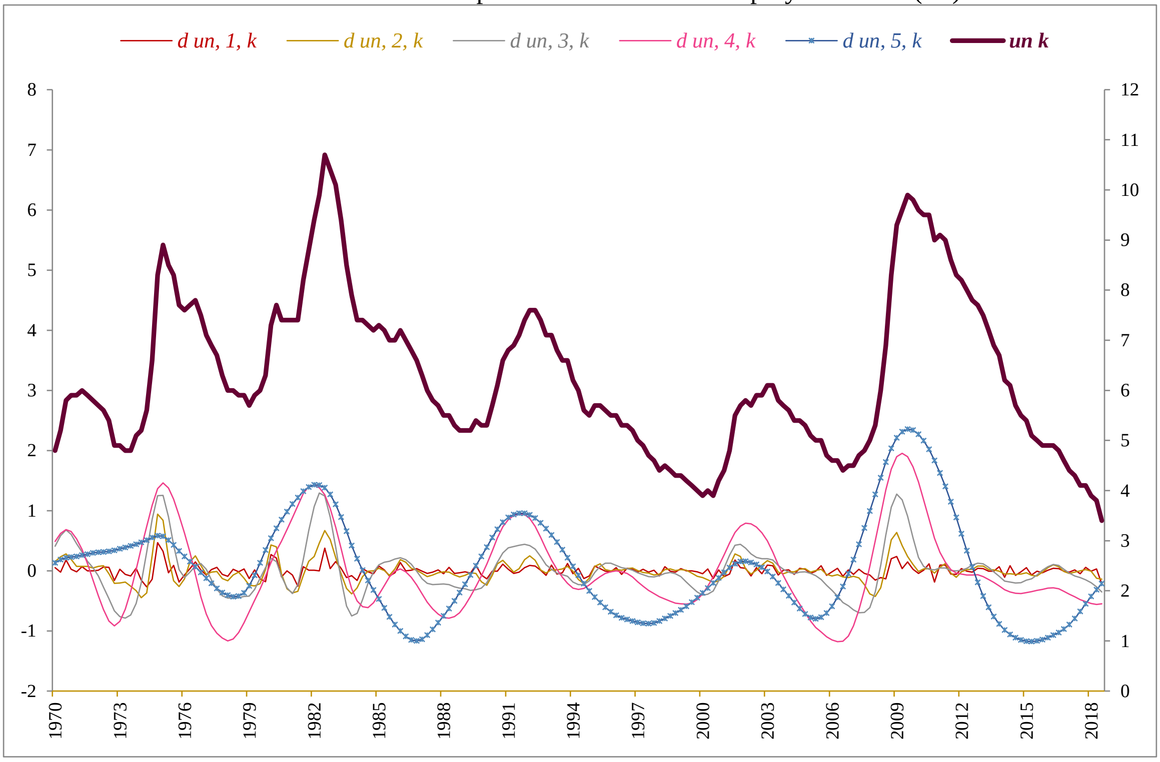
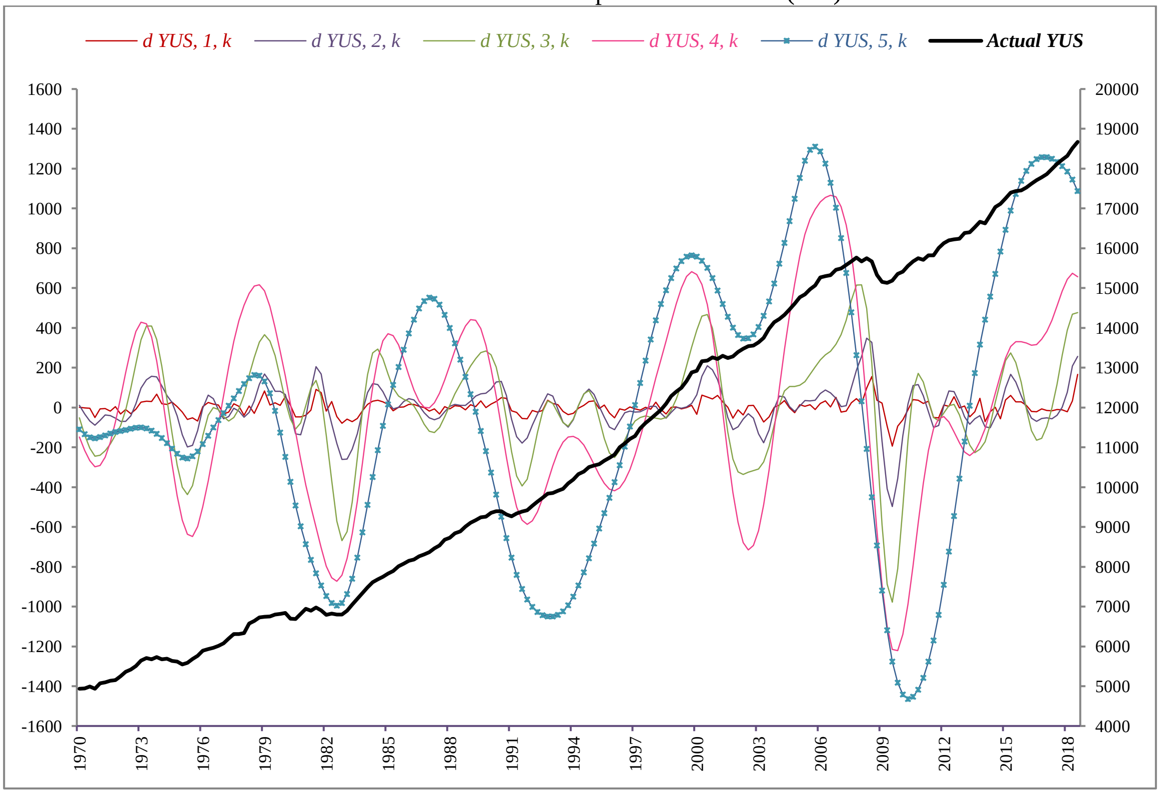


Figure 2

United States Wavelet Decomposition for GDP (Y^{US})



In figure 2 we plot the wavelet decomposition for U.S. real GDP. Once again we observe that the 4 to 8 year cycle crystal (d_4) dominates up until roughly 1980 and then the 8 to 16 year cycle crystal (d_5) becomes dominant. Interestingly the d_5 crystal peaked in 2016 and is beginning to go into a downward phase, which signifies that a recession would be forthcoming. In comparing with Aguiar-Conraria, Martins, and Soares (2020), the results are qualitatively similar, with the transition date from the higher frequency mode to the lower frequency mode occurring a little later (in roughly 1982), but the power of the lower frequency fluctuations appears to fade through time to essentially disappear by 2015. In terms of the cause of this modal shift in power from medium term to longer term cycles, this was first identified by Crowley and Hughes Hallett (2015) and then Crowley and Hughes Hallett (2018) offer an economic explanation for this phenomenon based on the New Keynesian and Classical theoretical models.

The model in equations (3) through (14) utilizes the framework in Crowley and Hudgins (2019) for each frequency range, $j = 1, \ldots, 5$, as, where the $\beta_{j;0}$ coefficients are constants and $L_{(.)}$ denotes the number of lags for any given variable. Blocks ir^{US}_j and ir^f_j are wavelet decompositions of the short-term domestic (US) and foreign (G6) interest rates, respectively. Block RER_j is the wavelet decomposed real exchange rate (index of foreign currency unit per US dollar), and the $\omega_{(.),j}$ terms represent blocks of random disturbance errors. Equation (3) specifies the consumption block as linearized functions of the lag structures of consumption, expected and lagged government spending, and the real exchange rate (*RER*). Expected government spending (G^e) permits a rational expectations component whereby GDP is crowded-out at each frequency range by increases in the national debt (*DEBT*).

$$C_{j,k} = \beta_{C,j,0} + f_{C,j}(C_{j,k-1}, \ldots, C_{j,k-L_C}, G^e{}_{j,k-1}, G_{j,k-1}, \ldots, G_{j,k-L_G}, RER_{j,k-1}, \ldots, RER_{j,k-L_{RER}}) + \omega_{C,j,k-1} \quad (3)$$

Investment is determined by domestic GDP and interest rates in equation (4). Net exports are a function of the lag structures of net exports, domestic GDP (Y^{US}), foreign GDP (Y^f), and the *RER*, as given in equation (5). The *RER* in equation (6) then captures interest rate parity influences from domestic and foreign interest rates.

$$I_{j,k} = \beta_{I\,j,0} + f_{I,j}(Y^{US}{}_{j,k-1}, \ldots, Y^{US}{}_{j,k-L_C}, ir^{US}{}_{j,k-1}, \ldots, ir^{US}{}_{j,k-L_{irUS}}) + \omega_{I,j,k-1} \quad (4)$$

$$NX_{j,k} = \beta_{NX,j,0} + f_{NX,j}(NX_{j,k-1}, \ldots, NX_{j,k-L_{NX}}, Y^{US}{}_{j,k-1}, \ldots, Y^{US}{}_{j,k-L_{Y^{US}}}, Y^f{}_{j,k-1}, \ldots, Y^f{}_{j,k-L_{Y^f}}, RER_{j,k-1}, \ldots, RER_{j,k-L_{RER}}) + \omega_{NX,j,k-1} \quad (5)$$

$$RER_{j,k} = \beta_{RER,j,0} + f_{RER,j}(ir^{US}{}_{j,k-1}, \ldots, ir^{US}{}_{j,k-L_{irUS}}, ir^f{}_{j,k-1}, \ldots, ir^f{}_{j,k-L_{irf}}, RER_{j,k-1}, \ldots, RER_{j,k-L_{RER}}) + \omega_{RER,j,k-1} \quad (6)$$

The unemployment rate is determined by OL at each frequency range, as specified in equation (7). In this equation, $\beta_{un,1} < 0$, so that the level of unemployment is a decreasing function of the domestic national output gap.

$$un_{j,k} - un^*{}_{j,k} = \beta_{un,0} + \beta_{un,1}\ (\%\Delta Y_{j,k} - \%\Delta Y^*{}_{j,k}) + \omega_{un,j,k} \quad (7)$$

In the wavelet decomposition of the data, OL can alternatively be captured by specifying the relationship in terms of the momentum of the detail crystals, as shown in equation (8):

$$d_{un,j,k} = \beta_{un,0} + \beta_{un,1}\, d_{y,j,k} + \omega_{un,j,k} \tag{8}$$

Equation (8) balances between the unemployment and output gaps version of OL in equation (7), and the percentage change version where $\Delta\, un_{j,k} = \beta_{un,0} + \beta_{un,1} \%\, \Delta\, Y_{j,k}$. The empirical results for equations (7) and (8) are given in appendix tables A1 and A2, respectively. Both specifications have adequate results, where the signs of all coefficients are as theoretically expected at all frequency ranges.[3]

The coefficient of determination is consistently larger for the estimated equations in equation (8). Based on the better empirical fit of equation (8) in table A2, the simulation analysis utilizes this version of OL. However, our WBC can equivalently employ the equation (7) and (8) specifications, and can accommodate both time-varying OL coefficients and the fixed OL coefficient specifications used here. The emphasis is on simulating the preliminary effects of economic policy in the WBC, so these results are only meant to employ reasonable values for the OL equation. We have also performed robustness checks and simulated the results for other values of the OL equations, but the results are not reported here in order to limit the length of the manuscript.

Inflation (*inf*) is determined in equation (9) by the inflation lags, the GDP gap, money supply growth (*MS*), and the *RER*. Since the Fed primarily utilizes the interest rate as an operating target, equation (10) determines the real money growth by adjusting to the lags of the real interest rate, the output gap, and real money growth, where the target inflation rate is inf^*= 2%.[4]

$$\begin{aligned} inf_{k} = {} & \beta_{inf,0} + \beta_{inf,1}\, inf_{j,k-1} + \beta_{inf,2}\,(Y_{k-1} - Y^*_{k-1}) + \beta_{inf,3}\, RER_{k-1} \\ & + \beta_{inf,4}\, MS_{k-1} + \beta_{inf,5}\, inf_{k-2} + \omega_{inf,k-1} \end{aligned} \tag{9}$$

$$\begin{aligned} MS_{k} - inf_{k} = {} & \beta_{MS,0} + \beta_{MS,1}\,(ir^{US}_{k-1} - inf_{k-1}) + \beta_{MS,2}\,(Y_{k-1} - Y^*_{k-1}) \\ & + \beta_{MS,3}\,(MS_{k-1} - inf_{k-1}) + \beta_{MS,4}\,(MS_{k-2} - inf_{k-2}) \\ & + \omega_{inf,k-1} \end{aligned} \tag{10}$$

The model is closed by equations (11) through (14), which contain the national income identity, passively determined net taxes (*T*), the quarterly budget deficit (*DEF*), and the debt stock.

$$Y_k = C_k + I_k + G_k + NX_k \tag{11}$$

[3] See the appendix for a more extensive commentary.

[4] The FOMC noted that an inflation rate of 2 percent (as measured by the annual change in the price index for personal consumption expenditures, or PCE) is most consistent over the longer run with the Federal Reserve's statutory mandate." Dec 19, 2018, Federal Reserve. For discussions and modification of the Taylor rule, see https://www.brookings.edu/blog/ben-bernanke/2015/04/28/the-taylor-rule-a-benchmark-for-monetary-policy/. Our model uses interest rates on short-term US Treasury securities (3-month T-bill rates), which follow the Fed Funds rates closely. See the Fed data for details at https://www.stlouisfed.org/on-the-economy/2017/october/increases-fed-funds-rate-impact-other-interest-rates.

$$T_k = \tau Y_k \tag{12}$$

$$DEF_k = G_k - T_k \tag{13}$$

$$DEBT_k = 0.25 DEF_k + (1 + i_k) DEBT_{k-1} \tag{14}$$

The fiscal policymakers choose government spending while the central bank chooses the interest rate at each frequency range in order to minimize the expected value of a quadratic performance index consisting of the weighted tracking errors for the variables of the model. Let x denote a state vector, and u denote a policy vector. Define the (*) as the target for any given variable, and let the superscript ^{T} represent the matrix transpose. The objective is to minimize the quadratic tracking index in expression (15):

$$\min_u E[J(u)] = (x_{K+1} - x^*_{K+1})^T Q_f (x_{K+1} - x^*_{K+1}) \tag{15}$$

$$+ \sum_{k=1}^{K} \left[(x_k - x^*_k)^T Q_k (x_k - x^*_k) + (u_k - u^*_k)^T R_k (u_k - u^*_k) \right]$$

The three terms in (15) penalize the policymakers for the tracking errors in the final state vector (with penalty matrix Q_f), the state vector in each period (with penalty matrix Q_k), and the control vector (with penalty matrix R_k). Following the methods of Crowley and Hudgins (2019), this determines the optimal simulated values for the 10 control variables and the 138 state variables in the large-scale WBC model.

4. Simulation Analysis

We use the coefficients from the estimated the model for the post-Bretton-Woods period of 1973 quarter 3 to 2018 quarter 2 reported in Crowley and Hudgins (2019), which provides satisfactory empirical results. The initial values for the simulations set the state variables in period 1 to correspond to the U.S. and foreign data in 2018, quarter 2. The unemployment rate target is set at 4.5%. The annual target growth rates for all real GDP variables are set at 2.5%. The 2% target inflation rate, combined with the targeted real GDP growth, results in a 4.5% annual nominal GDP growth target, which is consistent with a 4.5% money growth target. Given the 2% real interest rate target, the nominal interest rate is 4%.[5] Given that the initial nominal interest rate was only 2%, the unrestricted simulations specify approximate the Fed's "liftoff" strategy, as in Crowley and Hudgins (2018b), where the target annual interest rate is initially 2%, but steadily increases over the horizon, where it achieves a final value of 4%.[6]

The simulations in this analysis all assume political cycle targeting as in Crowley and Hudgins (2015), with the primary emphasis being on the cycles between 2 and 8 years. The main purpose of the simulations is to analyze the relative changes

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[5] This balances a real interest rate of 2% with a productivity growth of 2%. For an annual population growth of 0.5%, this is consistent with an annual real GDP target growth of 2.5%.

[6] The target interest rate is thus growing at a quarterly compounded growth rate of 0.04729. This approximates an interest rate response in the short-term bond market to series of eight semi-annual Fed discount rate increases by 25 basis points over the four-year horizon.

in the optimal forecast trajectories that occur when the fiscal and monetary policy prioritize the following cases:

(1) the unemployment gap, which is the base case;
(2) the output gap, where the priority is on economic growth, and
(3) the inflation gap, where the priority is on an inflation target.

As shown in Table 2, we conducted simulations under different initial economic conditions reflecting each phase of the business cycle. During the latter part of 2018, the U.S. was experiencing both low initial unemployment and low inflation as it was widely thought to be in the latter part of the expansionary stage of the business cycle. Figures 3 – 5 shows the simulation results for cases (1), (2), and (3), respectively, for this initial late-cycle phase, which incorporates the actual initial U.S. data for all of the variables. The cumulative summary results for the initial conditions are shown in Table 3. Table 3 presents the results cumulative percentage changes of case (2) and case (3) relative to case (1) for each of these sensitivity scenarios. For sensitivity analysis in all cases, we simulated the model for 2 penalty scenarios: (A) large penalties for final period tracking errors of the aggregate unemployment and GDP component state variables, and (B) small penalties for final state tracking errors. Figures 3 – 5 illustrate the forecasts for penalty scenario (Aa).

Table 4 shows the cumulative results when the economy is initially in a recession, with high initial unemployment and low inflation. Table 5 shows the cumulative results when the economy is initially experiencing an inflationary gap, with low initial unemployment and high inflation. Finally, Table 6 shows the cumulative results when the economy is initially experiencing stagflation, with high initial unemployment and high inflation.

Table 2

Initial Economic Condition Specifications for Simulated Forecasts

	Low Initial Inflation	*High Initial Inflation*
Low Initial Unemployment	Late Cycle Phase Table 3 Figures 3 – 5	Inflationary Gap Table 5
High Initial Unemployment	Recession Table 4	Stagflation Table 6

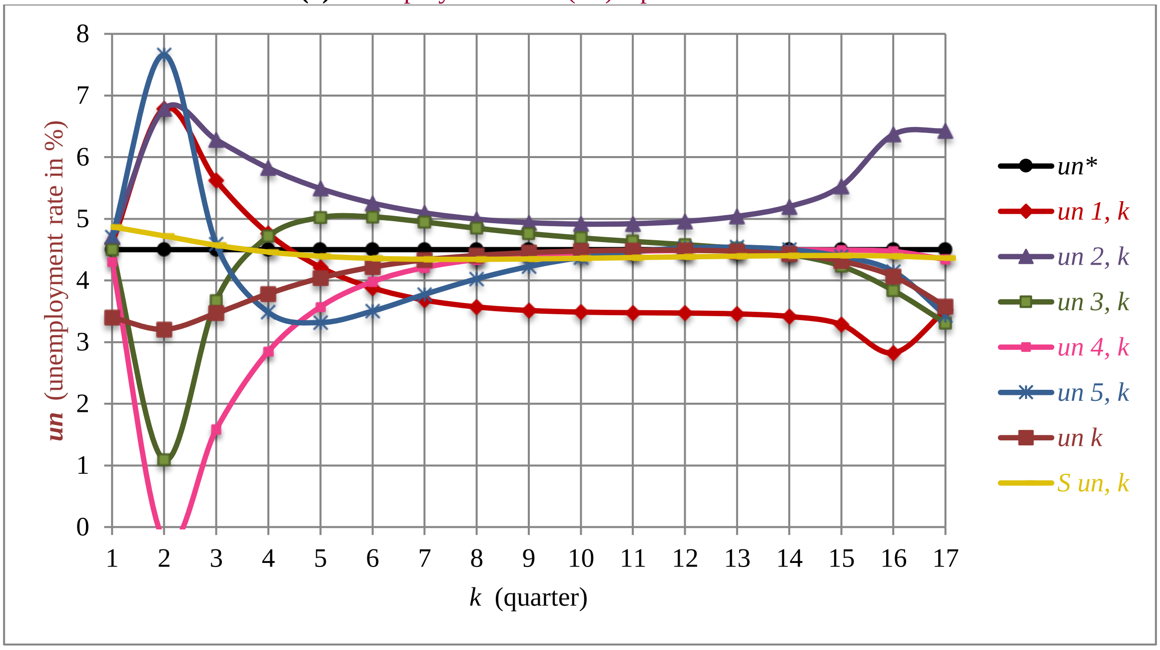
In the simulations, at each frequency range, government spending and the nominal interest rate is set at their actual 2018 quarter 1 values in period 0. The fiscal authorities choose government spending, while the central bank chooses the nominal interest rate, beginning in quarter 1 and ending four years later in quarter 16. The state variables are set at their initial 2018 quarter 2 values in period 1, and the forecasts begin in quarter 2, and end in quarter 17.

Both government spending and the interest rate trajectories are determined simultaneously by the single performance index in (15), which governs fiscal and monetary policy through optimal feedback control. Thus, the relative tracking error penalty parameter weights in equation (15) must be the same for both fiscal and monetary policy. Although Crowley and Hudgins (2020) alter the problem specification so that monetary policy is determined by a TR, the feedback control trajectories are still determined by a single performance index, which cannot be decoupled for fiscal and monetary policy. However, the user can simulate the results under different sets tracking penalty parameters that reflect a different relative joint policy preferences for monetary and fiscal policy.

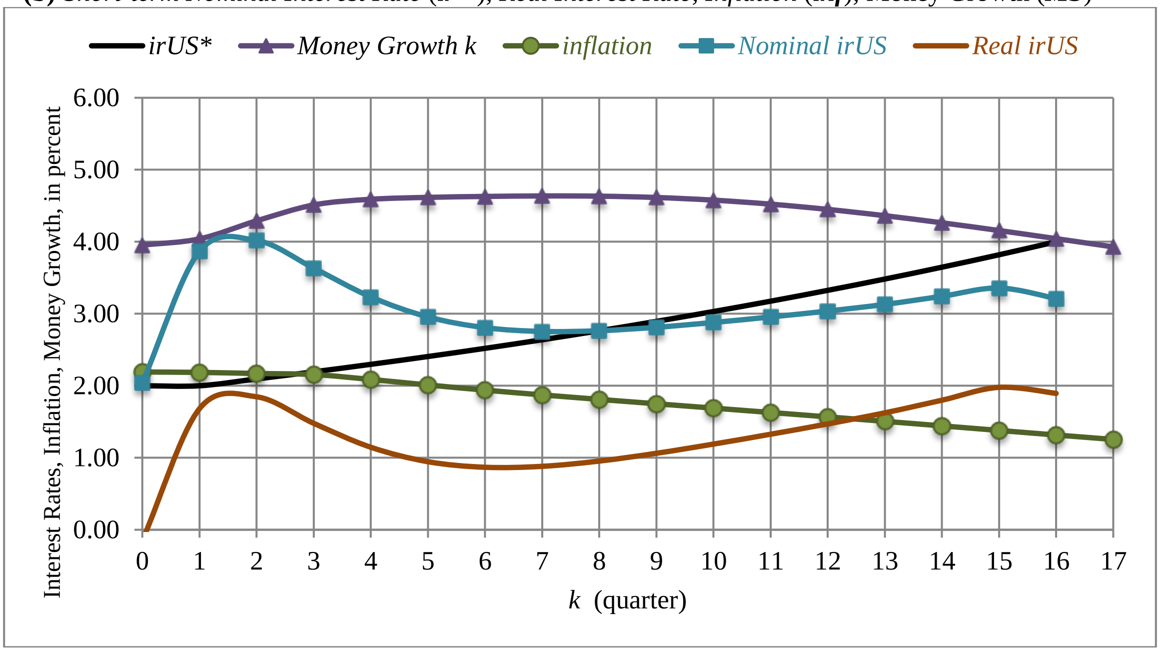
Figure 3

Case (1): Small Weight on *National Output* Tracking Error

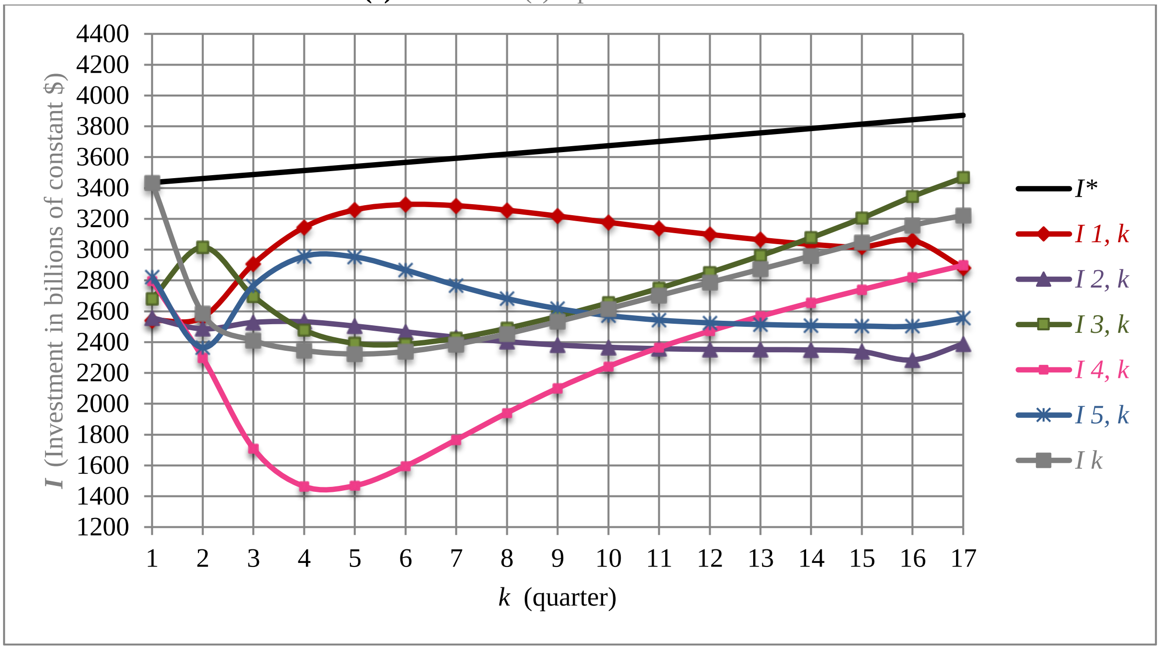
(a) Unemployment Rate (***un***) Optimal Forecasts



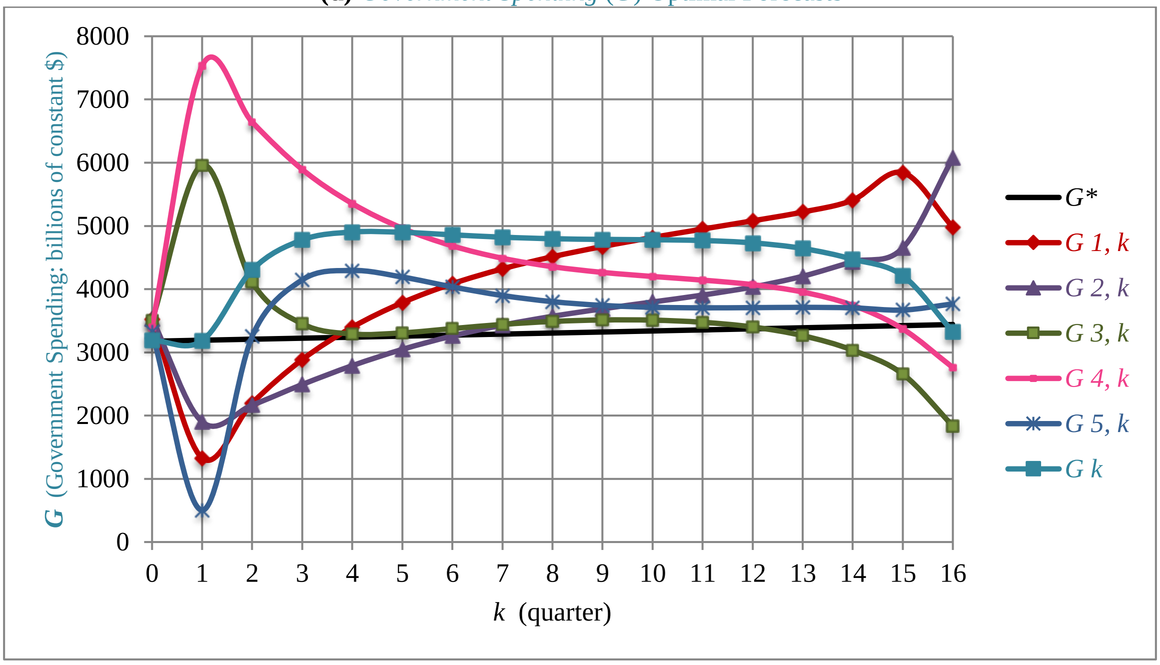
(b) *Short-term Nominal Interest Rate* (**ir^{US}**), *Real Interest Rate*, *Inflation* (***inf***), Money Growth (***MS***)



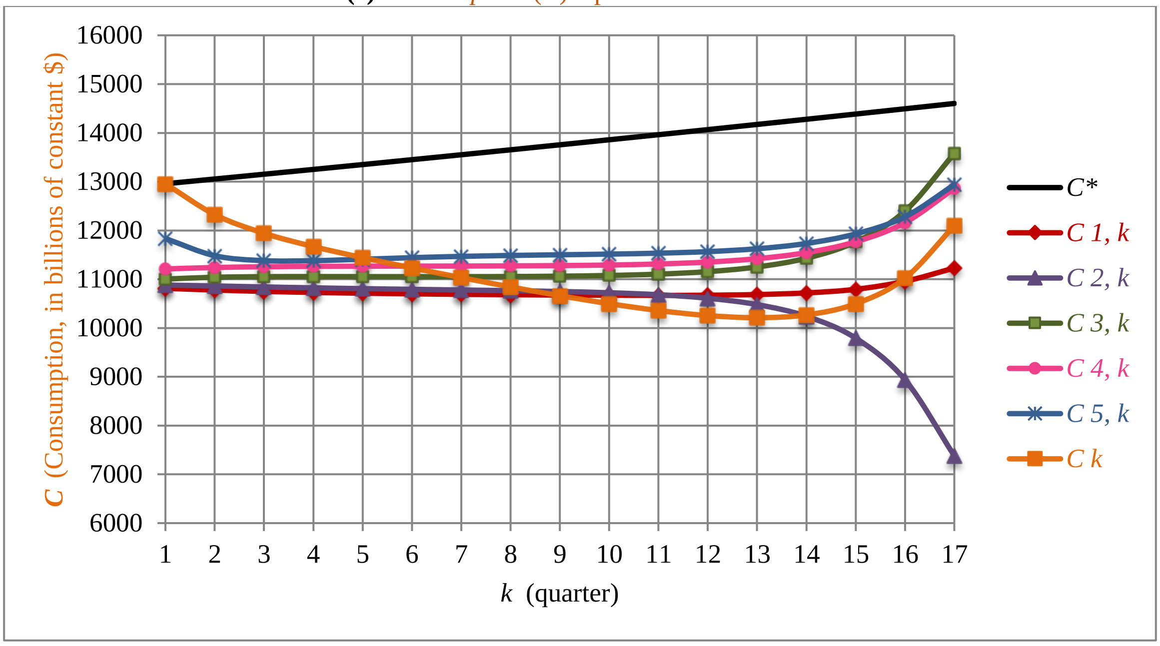
(c) *Investment* (***I***) Optimal Forecasts



(d) *Government Spending* (***G***) Optimal Forecasts



(e) *Consumption* (***C***) Optimal Forecasts



(f) *National Output* (***Y***) Optimal Forecasts

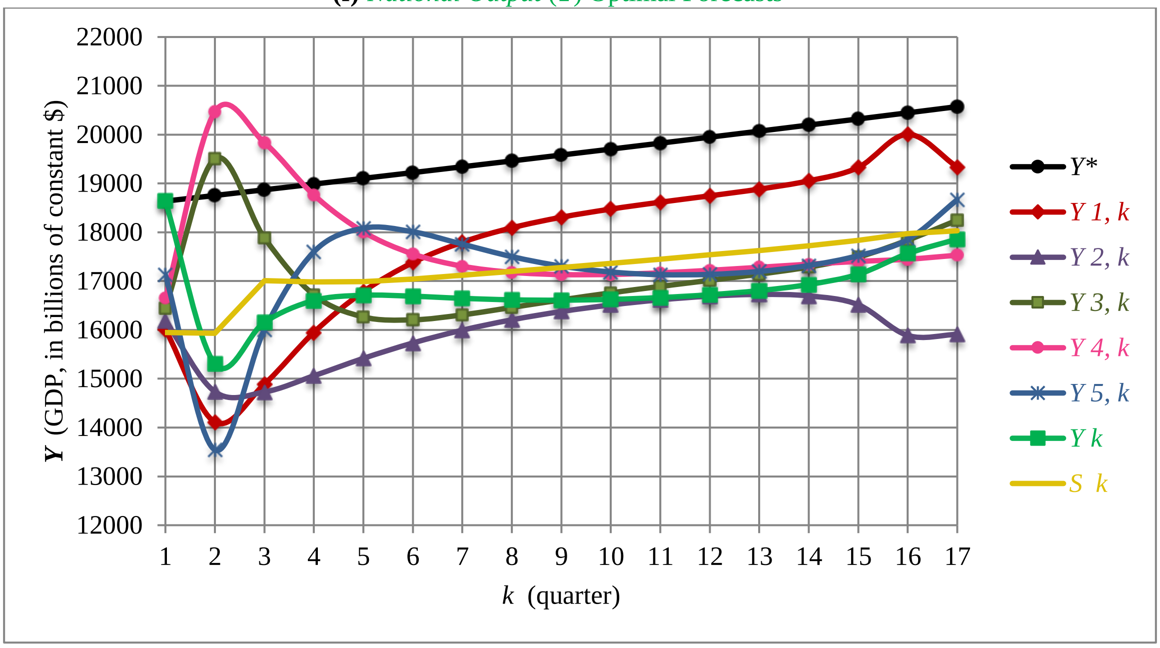


Figure 3 shows the base case where the unemployment tracking error receives a relatively large weight. Figure 3(a) shows the forecast trajectories for the unemployment rate, where the aggregate unemployment rate begins below its target. Since national output growth initially begins to lag below its target, government spending increases above its target, especially at the political cycle frequency ranges 3 and 4 (from 2 to 8 years). This fiscal stimulus increases output, causing a decrease in unemployment through the OL mechanism at these frequency ranges. The negative detail crystals for unemployment at frequency range 3 and 4 dominate the positive crystals at the other frequency ranges, causing aggregate unemployment to drop initially.

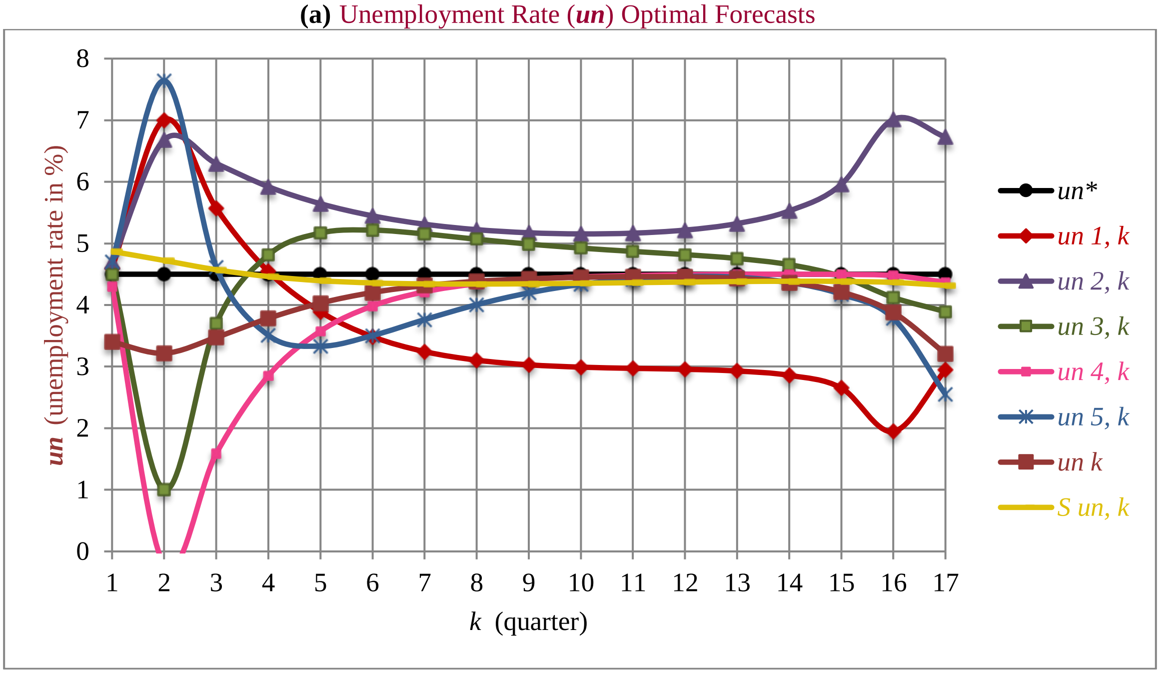
Monetary policy is contractionary so that it counteracts the unsustainably low unemployment rate, and expansionary fiscal policy's downward pressure on unemployment. The interest rate increases in quarters 1 through 3 to reach 4%, before declining throughout the middle of the horizon. This initial interest rate increase causes a decline in investment, and it causes an increase (appreciation) in the RER that has a downward effect on net exports. Thus, aggregate consumption and GDP falls short of their targets. The contractionary monetary policy also results in money growth that slightly increases to reach a level maximum at about 4.63% for the first half of the horizon, before maintaining a slight steady decline for the remaining quarters. This results in a steady fall in inflation rates, and the real interest rate never falls below 0.87%.

In the middle of the horizon, government spending at frequency ranges 2, 3, and 5 are close to the target, while aggregate government spending begins to level and then decrease. This balances monetary policy so that the unemployment rate closely tracks its target in the middle periods, both in the aggregate and at the political cycle frequency ranges 3 and 4. At the end of the horizon, the nominal interest rate begins to track slightly below its target, while government spending decreases to end at a value that is close to its target. These monetary and fiscal variables contribute to an increase in GDP and all of its components. The slight increases in GDP causes aggregate unemployment to end the horizon slightly below its target.

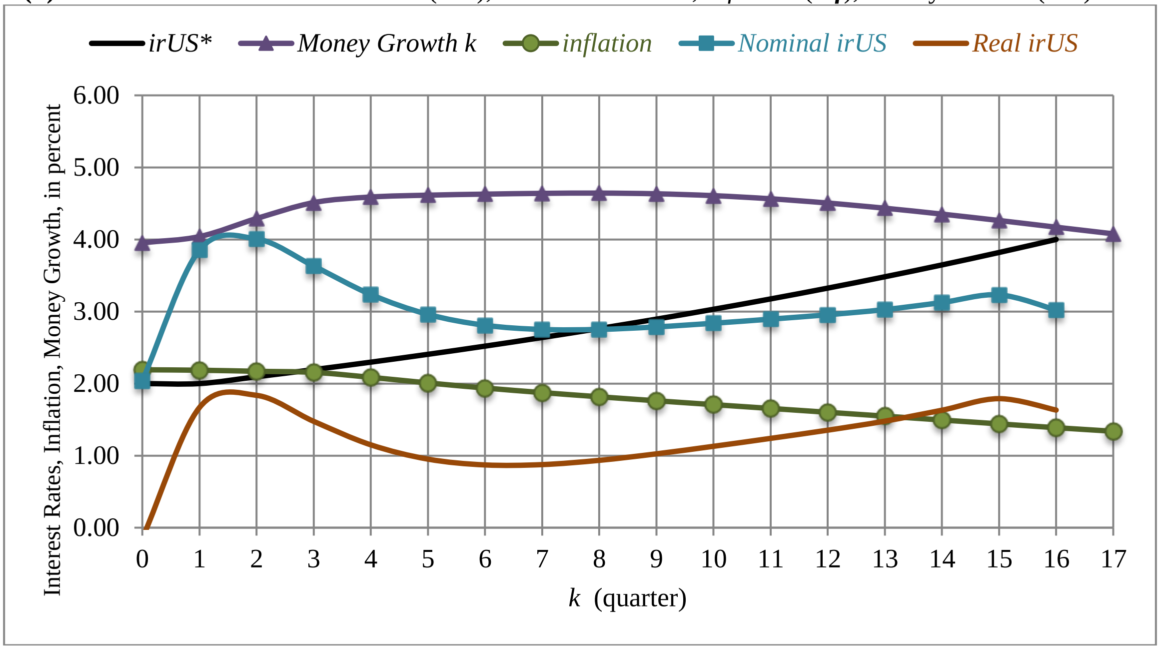
Figure 4

Case (2): Large Weight on *National Output* Tracking Error

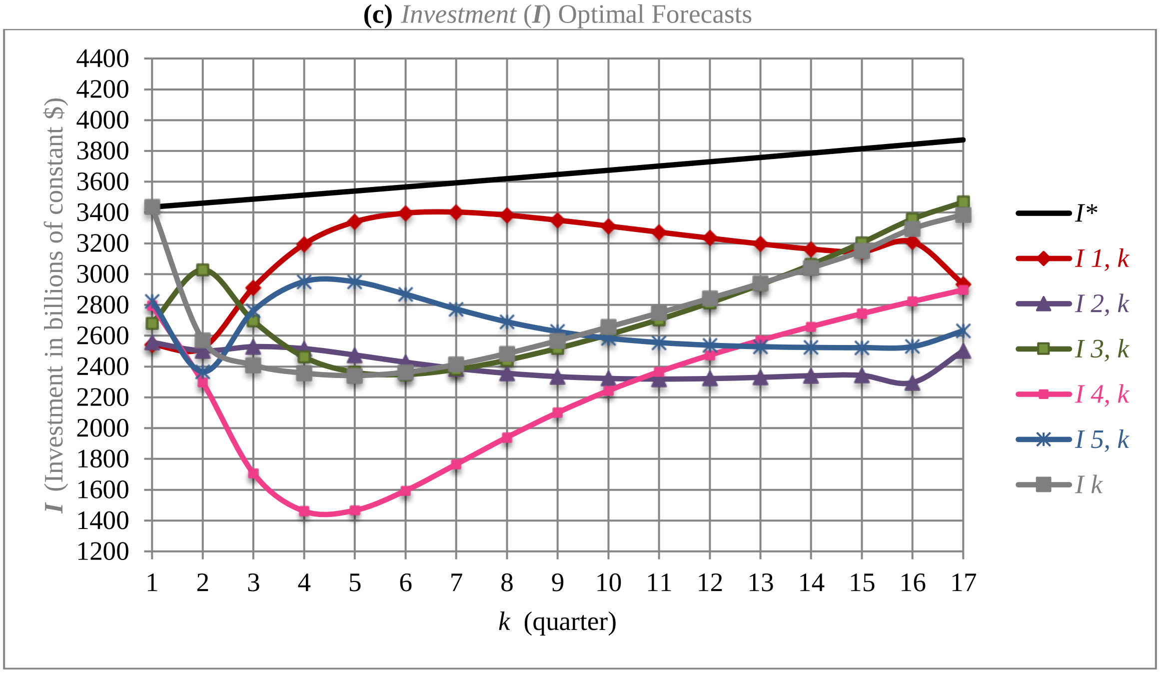
(a) Unemployment Rate (***un***) Optimal Forecasts



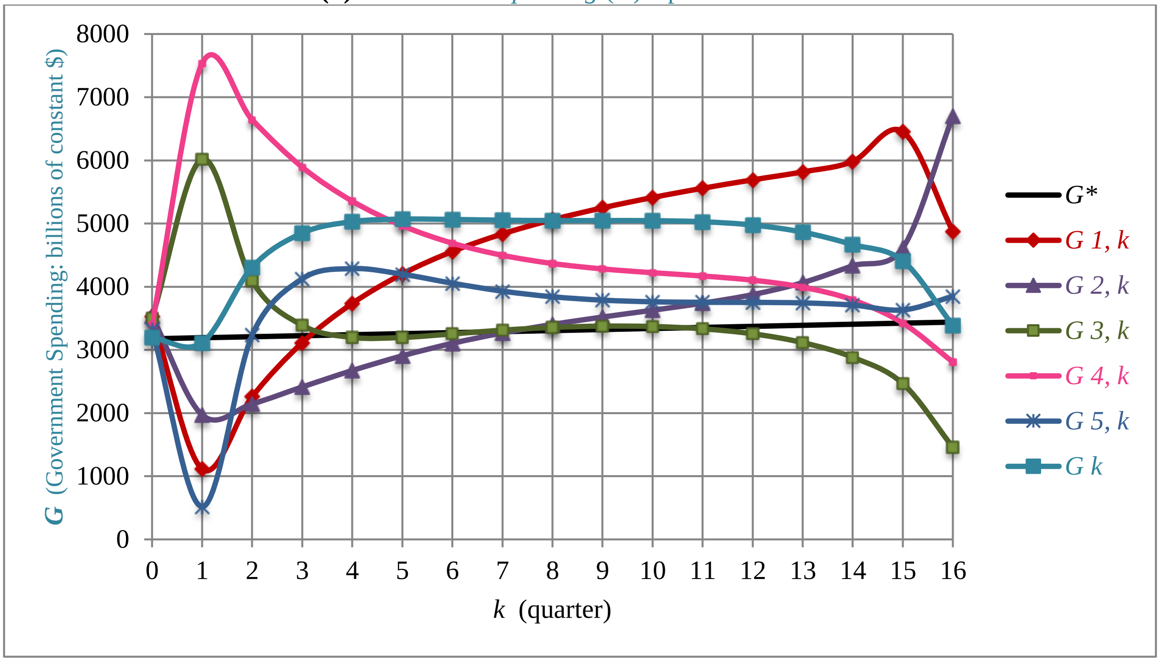
(b) *Short-term Nominal Interest Rate* (***ir***^{US}), *Real Interest Rate, Inflation* (***inf***), Money Growth (***MS***)



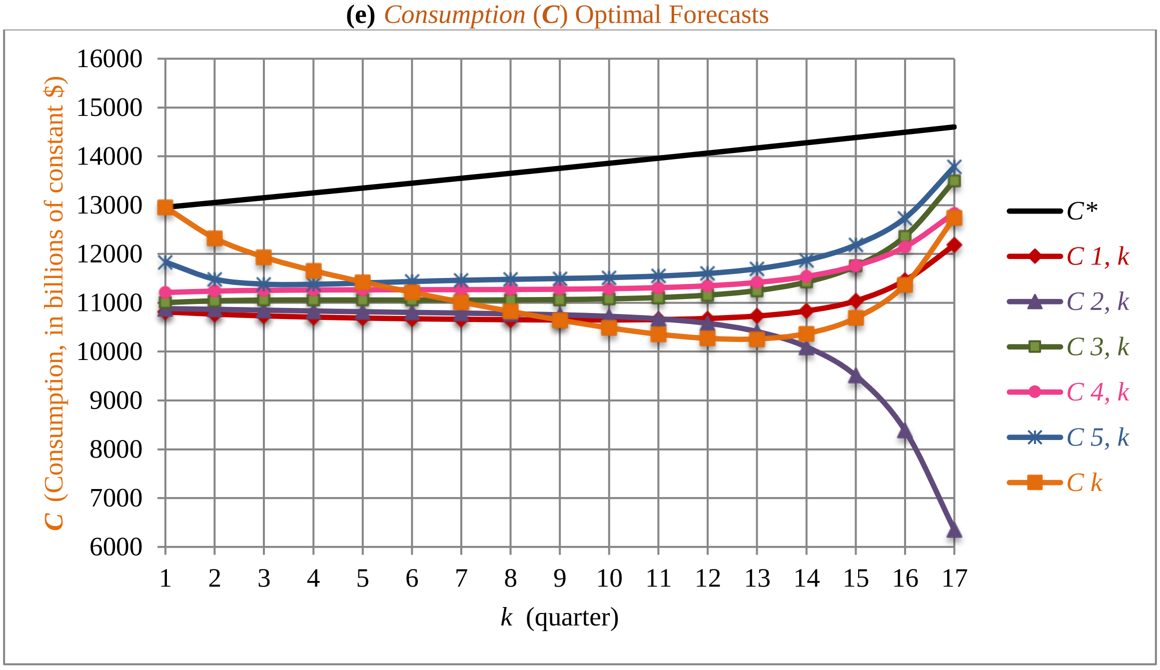
(c) *Investment* (***I***) Optimal Forecasts



(d) *Government Spending* (***G***) Optimal Forecasts



(e) *Consumption* (***C***) Optimal Forecasts



(f) *National Output* (***Y***) Optimal Forecasts

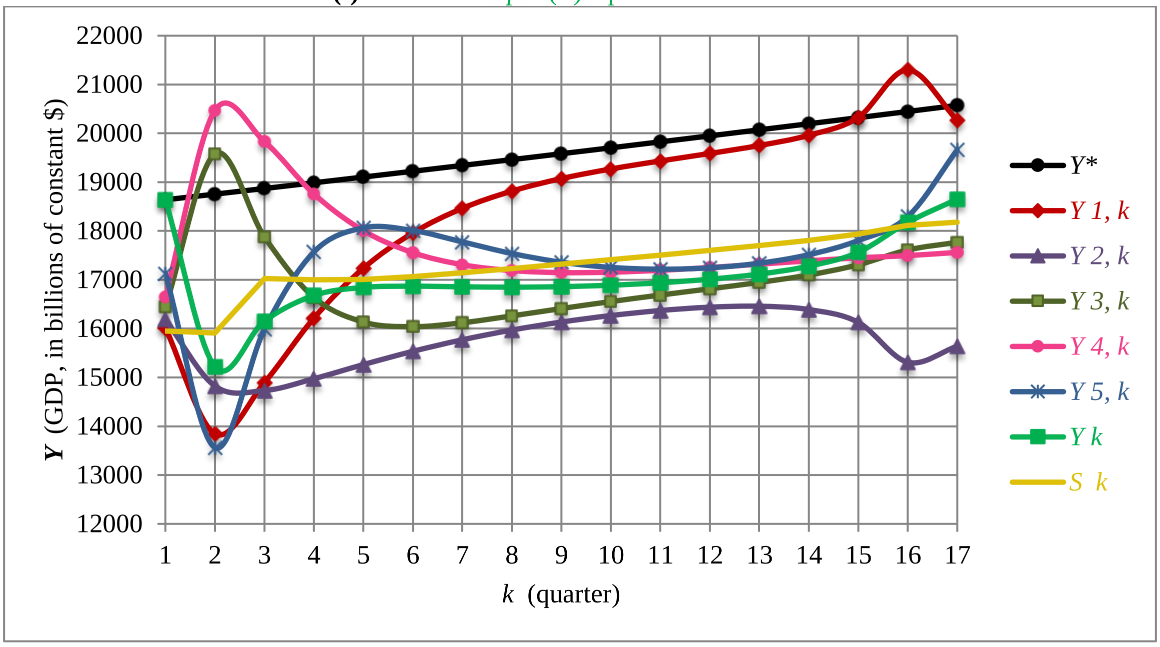


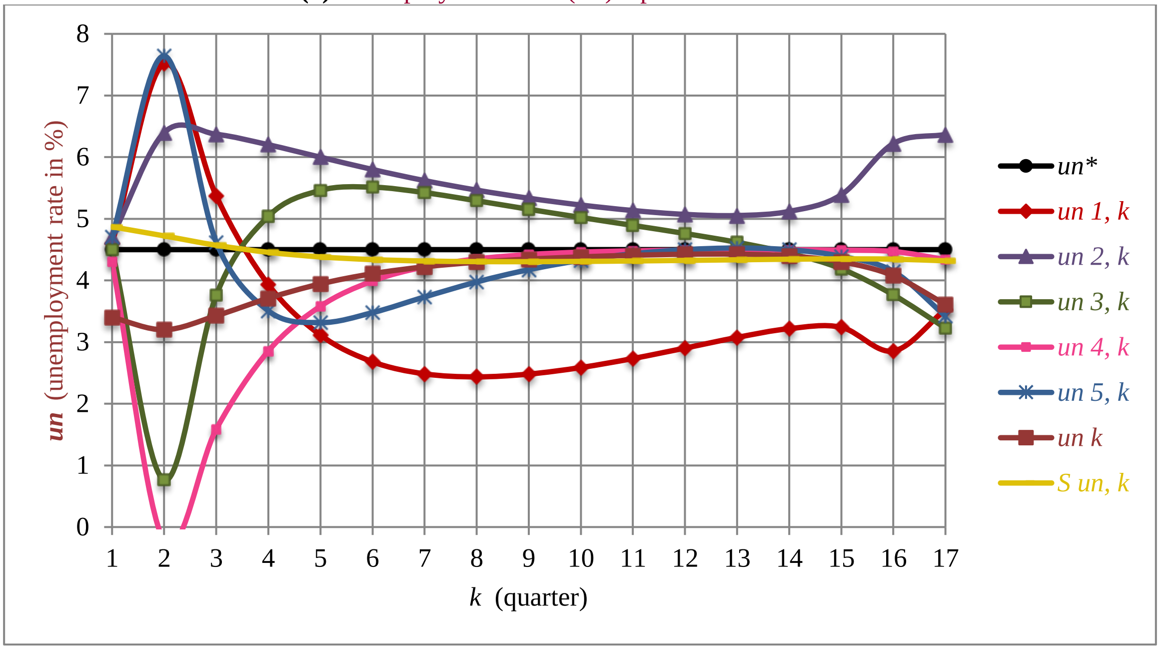
Figure 4 shows case (2) where economic growth tracking received a relatively large weight compared to the unemployment rate and inflation. The aggregate unemployment rate trajectory in Figures 3(a) and 4(a) are similar. However, the frequency range 1 (6 months to 1 year) unemployment trajectory is lower in figure 4 than in Figure 3, and the frequency 2 (1 to 2 years) trajectory higher in figure 4 than in figure 3. This is mirrored by the GDP components, where the figure 4 trajectories for frequency range 1 are much higher than those in case (1), and the figure 4 trajectories for frequency range 2 are much lower than those in case (1). Thus, there is a shift in case (2) toward expansionary policy at the shortest cycle.

Table 3 shows that monetary and fiscal policy are more expansionary in case (2) when there is increased emphasis on economic growth. In penalty scenario (1) for case (2), the cumulative aggregate unemployment level is 1.33% lower than in case (1), which shows that the policymakers are willing risk an overheating economy with the potential for wage and price inflation. Cumulative aggregate government spending is 3.71% higher in case (2), showing that fiscal policymakers are willing to incur more debt when there is increased emphasis on economic growth. Monetary policy is also more expansionary in case (2), where the cumulative aggregate interest rate is 1.48% lower and cumulative money growth is 0.89% higher than in case (1). As a result, cumulative investment is 1.80% higher in case (2) than in case (1), and cumulative aggregate output is 1.50% higher than in case (1).

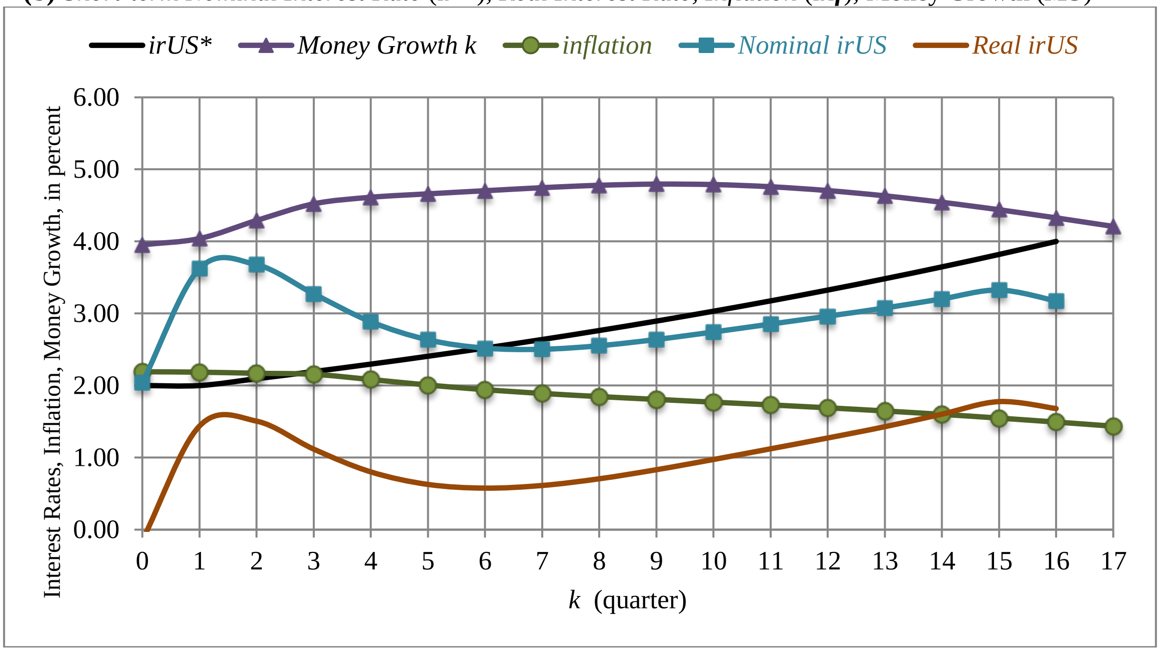
Figure 5

Case (3): Large Weight on *Inflation* Tracking Error

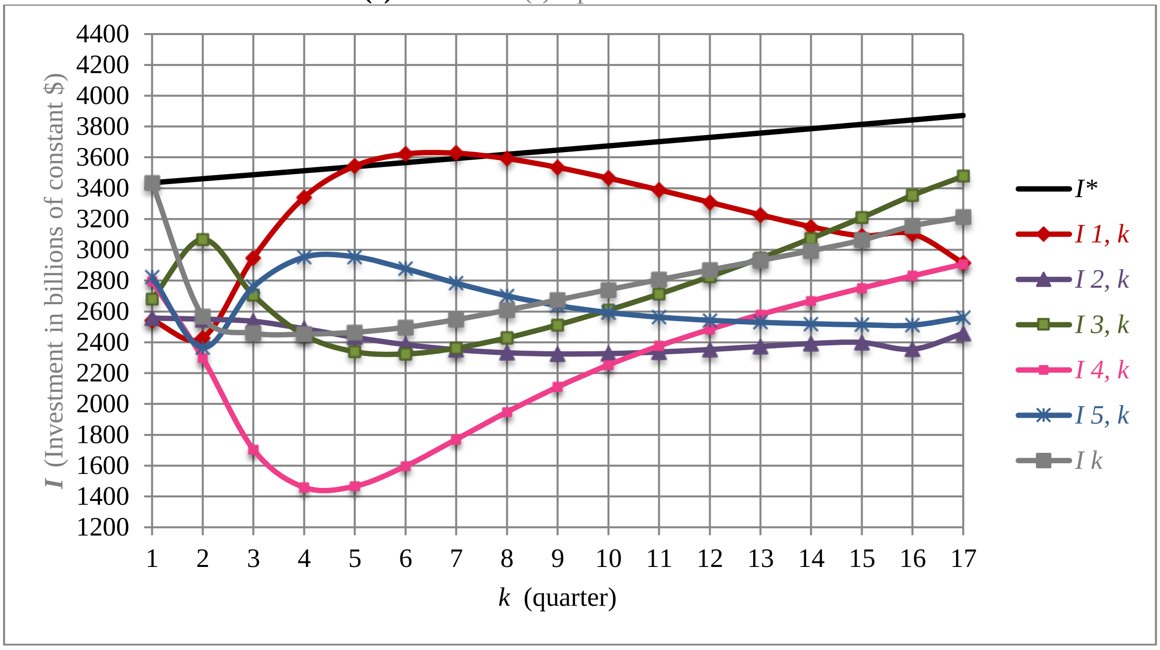
(a) Unemployment Rate (***un***) Optimal Forecasts



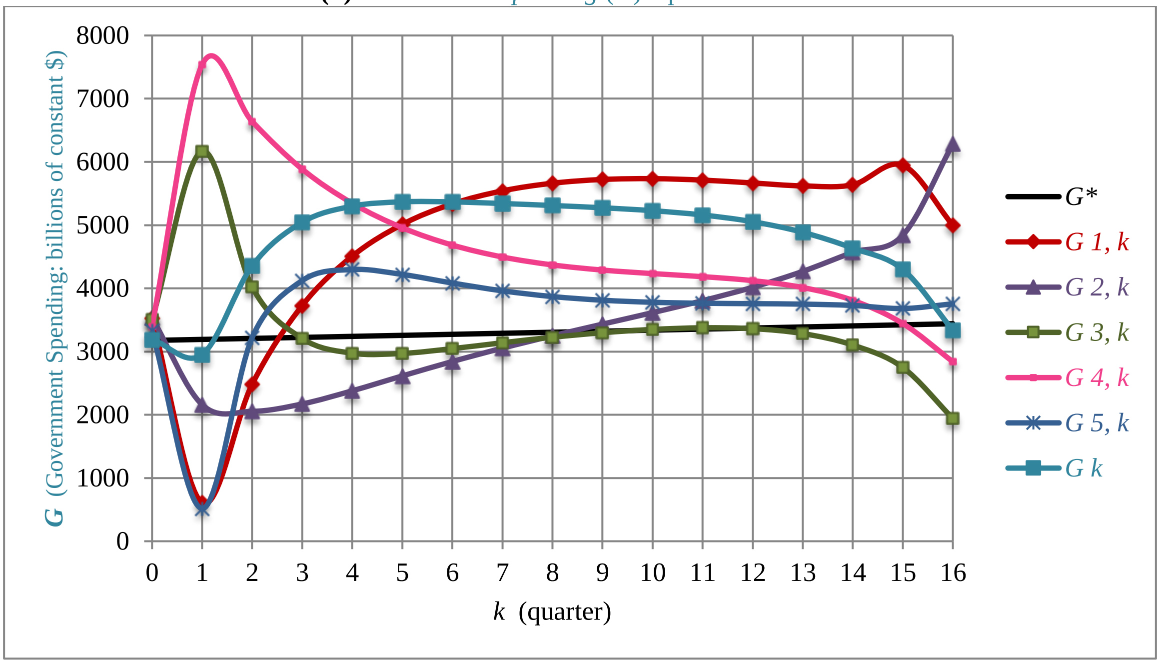
(b) *Short-term Nominal Interest Rate* (***ir**^{US}*), *Real Interest Rate, Inflation* (***inf***), Money Growth (***MS***)



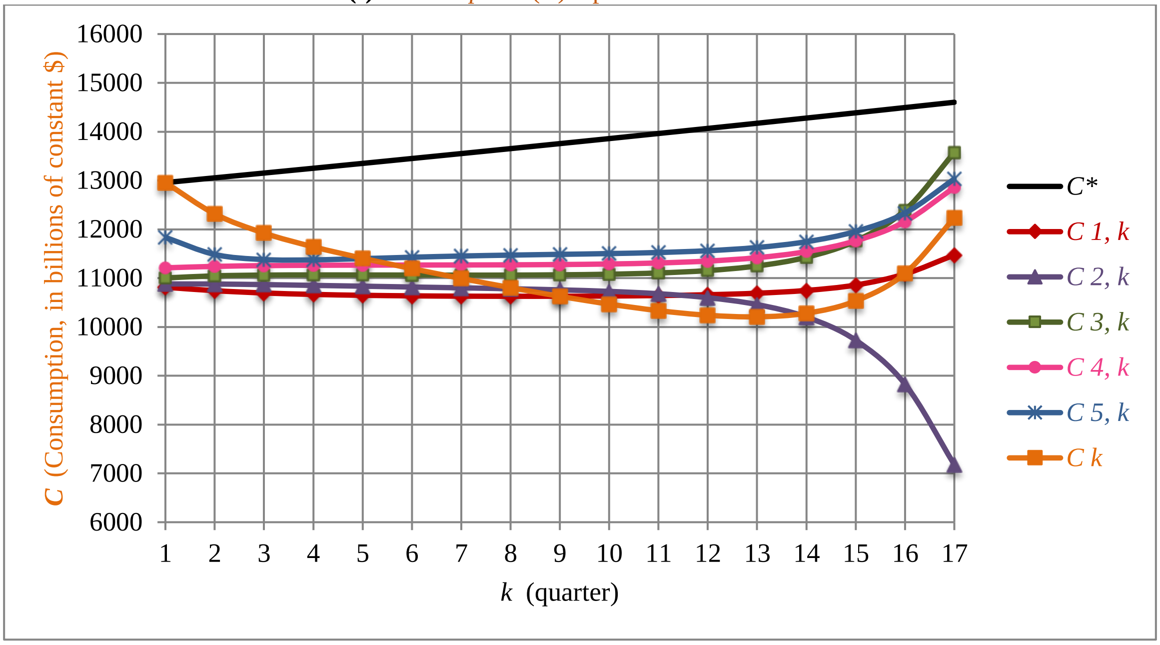
(c) *Investment* (***I***) Optimal Forecasts



(d) *Government Spending* (***G***) Optimal Forecasts



(f) *Consumption* (***C***) Optimal Forecasts



(f) *National Output* (***Y***) Optimal Forecasts

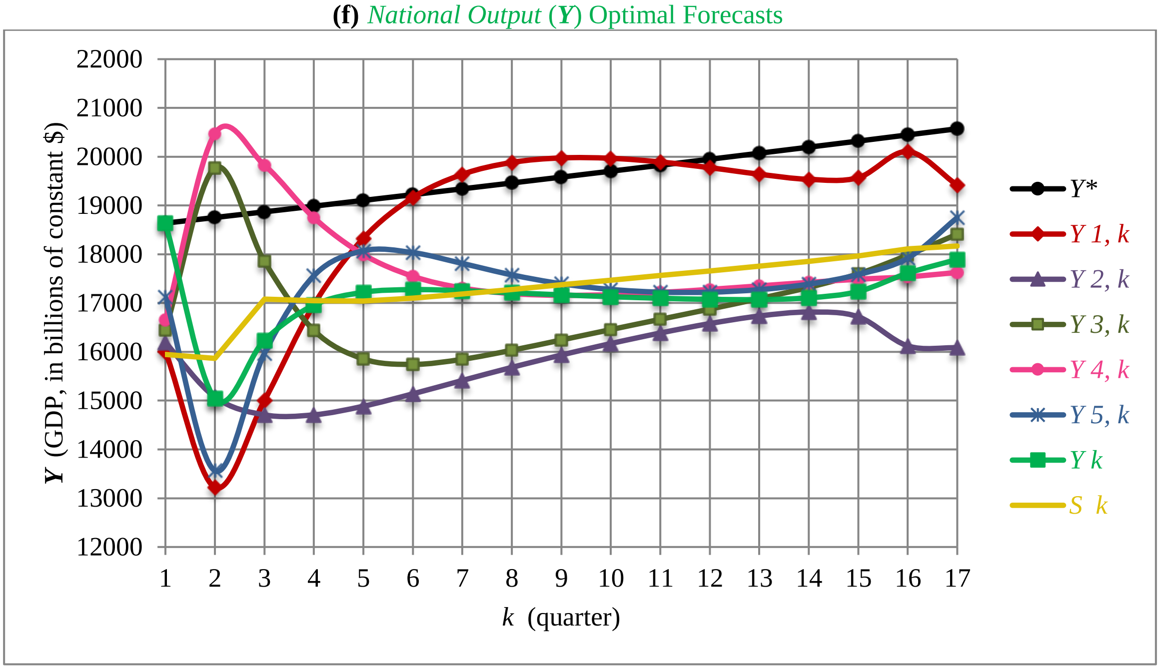


Figure 5 shows case (3) where the inflation rate received a relatively large weight compared to the unemployment rate and economic growth tracking. Given that inflation and output growth are initially low, the policy emphasis is expansionary. Here in case (3), monetary policy is much more aggressively expansionary than in cases (1) and (2). In penalty scenario (2), cumulative money growth is 29.04% greater in case (3) than in case (1), and the cumulative aggregate interest rate summation 44.78 lower in case (3) than in case (1). In penalty scenario (A), cumulative money growth is 3.41% greater in case (3) than in case (1), and the cumulative aggregate interest rate summation

5.72% lower in case (3) than in case (1). This lower interest rate leads to a 2.83% increase in cumulative aggregate investment in case (3) versus case (1).

The case (3) lower interest rate leads to a lower (depreciated) RER trajectory which puts downward pressure on net exports, but this effect is overwhelmed by the GDP increase that causes imports to increase and net exports to fall. In penalty scenario (B), the overall effect is that cumulative net exports in case (3) are 8.45% lower than in case (1). The inflation trajectory in figure 5 tracks its target much closely than in the previous cases, and in penalty scenario (2), cumulative inflation is 3.87% higher in case (3) than in case (1).

When compared to monetary policy, fiscal policy is relatively less aggressive in case (3) than in the economic growth emphasis case (2). Table 3 shows that for penalty scenario (B), cumulative government spending is 1.31 % (5.30 – 6.61) less in case (3) and in case (2). This shows that it is optimal to more heavily utilize monetary policy to achieve the inflation target objective, whereas fiscal policy is the more heavily used when the relative policy emphasis is tracking an economic growth target. The increase in cumulative output of 5.60% in case (3) over case (1) results in a 4.43% decrease in cumulative aggregate unemployment through OL. This shows that policymakers might be more concerned with economic stimulation, and less concerned about labor market redistribution and productivity problems that might result from the unemployment rate lingering below its natural rate.

Table 3

Comparison of Cumulative Differences (in %) by Real GDP component under the increased prioritization of Economic Growth and Inflation Targeting objectives

Late Cycle Phase: Initial Condition Specified with Low Unemployment and Low Inflation

	Penalty Scenario (A)		**Penalty Scenario (B)**	
Variable	**% Change with Economic Growth Emphasis (large final tracking error penalty)**	**% Change with Inflation Emphasis (large final tracking error penalty)**	**% Change with Economic Growth Emphasis (large final tracking error penalty)**	**% Change with Inflation Emphasis (large final tracking error penalty)**
C	0.66	– 0.03	2.46	1.54
I	1.80	2.83	2.82	26.15
G	3.71	6.42	6.61	5.30
NX	– 1.81	– 3.82	– 2.86	– 8.45
Y	1.50	1.73	3.39	5.60
Ir	– 1.48	– 5.72	– 0.47	– 44.78
MS	0.89	3.41	1.80	29.04
Inf	1.26	3.87	2.47	18.03
Un	– 1.33	– 1.35	– 1.10	– 4.43

Under penalty scenario (A), fiscal policy is somewhat more aggressive in case (3) than in the economic growth emphasis in case (2). The cumulative government spending under penalty scenario (A) is 2.71% (6.42 – 3.71) more in case (3) and in case (2). This shows that in the late cycle phase, fiscal policy may also be more aggressive with an inflation target than with an economic growth target, but the relative increase in intensity is not as strong for fiscal policy as with the aggressive shift in monetary policy.

In order to test this result across the other phases of the business cycle, we also simulated the economy with counterfactual initial conditions. Leaving all else the same, Table 4 presents the cumulative aggregate variable comparisons for the simulations when the economy is initially in a recession with high unemployment, rather than the low initial unemployment of the late-cycle scenario. So to summarize, Table 4 compares the cumulative changes in each of the real GDP components over the forecast horizon under the increase in the prioritization of economic growth targeting and inflation targeting when compared with the base case that relatively emphasizes achieving an unemployment target.

Table 4

Comparison of Cumulative Differences (in %) by Real GDP component under the increased prioritization of Economic Growth and Inflation Targeting objectives

Recession: Initial Condition Specified with High Unemployment and Low Inflation

	Penalty Scenario (A)		**Penalty Scenario (B)**	
Variable	**% Change with Economic Growth Emphasis (large final tracking error penalty)**	**% Change with Inflation Emphasis (large final tracking error penalty)**	**% Change with Economic Growth Emphasis (large final tracking error penalty)**	**% Change with Inflation Emphasis (large final tracking error penalty)**
C	0.58	– 0.005	1.91	0.59
I	1.04	0.44	1.61	7.22
G	3.00	1.39	4.96	2.10
NX	– 1.30	-0.71	– 1.87	– 2.94
Y	1.18	0.35	2.47	5.60
ir	– 0.88	– 1.19	– 0.52	– 29.55
MS	0.55	0.56	0.88	7.77
inf	0.78%	0.59	1.29	5.02
un	– 0.93	– 0.21	– 0.67	– 1.35

Table 4 shows mostly the same patterns as Table 3. Under penalty scenario (1), cumulative aggregate government spending is 1.61% (3.00 – 1.39) higher under the economic growth emphasis when compared with the inflation emphasis. In penalty scenario (2), cumulative aggregate government spending is 2.86% (4.96 – 2.10) higher under the economic growth emphasis when compared with the inflation emphasis.

However, the aggregate interest rate trajectory is much lower when the inflation target is emphasized over the economic growth target. The cumulative aggregate interest rate is 0.31% (1.19 – 0.88) lower, and the money supply growth is 1% (0.56 – 0.55) higher, in case (3) when an inflation target is relatively more heavily emphasized. This shows that monetary policy is again more aggressively utilized to achieve an objective that prioritizes an inflation-based target.

We also simulate the counterfactual initial conditions where the economy has an inflationary gap with low unemployment and high inflation, and summarize the results in Table 5. For the inflationary gap, both fiscal and monetary policy are more expansionary when the priority is sustaining the economic growth occurring at peak of the economic cycle. However, under the inflation priority, both policies become contractionary in an attempt to reduce the overheating inflation. Monetary policy takes on an aggressively contractionary thrust where the interest rate trajectories are substantially above those in the other cases.

Table 5

Comparison of Cumulative Differences (in %) by Real GDP component under the increased prioritization of Economic Growth and Inflation Targeting objectives

Inflationary Gap: Initial Condition Specified with Low Unemployment and High Inflation

	Penalty Scenario (A)		**Penalty Scenario (B)**	
Variable	**% Change with Economic Growth Emphasis (large final tracking error penalty)**	**% Change with Inflation Emphasis (large final tracking error penalty)**	**% Change with Economic Growth Emphasis (large final tracking error penalty)**	**% Change with Inflation Emphasis (large final tracking error penalty)**
C	1.09	– 0.01	2.47	– 0.19
I	1.96	– 3.21	2.86	– 4.25
G	7.00	– 6.91	6.66	– 0.51
NX	– 2.67	4.29	– 2.89	1.35
Y	2.38	– 1.69	3.42	– 0.78
ir	– 0.47	6.01	– 0.46	7.11
MS	0.87	– 3.18	0.86	– 2.36
inf	1.05	– 2.81	0.82	– 1.07
un	– 2.23	1.97	– 1.10	0.80

Table 6

Comparison of Cumulative Differences (in %) by Real GDP component under the increased prioritization of Economic Growth and Inflation Targeting objectives

Stagflation: Initial Condition Specified with High Unemployment and High Inflation

	Penalty Scenario (A)		**Penalty Scenario (B)**	
Variable	**% Change with Economic Growth Emphasis (large final tracking error penalty)**	**% Change with Inflation Emphasis (large final tracking error penalty)**	**% Change with Economic Growth Emphasis (large final tracking error penalty)**	**% Change with Inflation Emphasis (large final tracking error penalty)**
C	0.74	0.01	1.92	– 0.32
I	1.31	– 4.84	1.63	– 4.86
G	4.19	– 12.01	5.01	– 1.00
NX	– 1.76	7.53	– 1.90	2.01
Y	1.56	– 3.06	2.49	– 1.19
ir	– 0.72	11.95	– 0.51	19.32
MS	0.55	– 4.98	0.50	– 3.08
inf	0.64	– 4.42	0.51	– 1.47
un	– 1.29	2.71	– 0.68	1.03

Table 6 summarizes the final counterfactual case where the economy is initially experiencing stagflation, with initial high unemployment and high inflation. Under stagflation, the policymakers must balance the conflicting forces of inflation reduction (which curbs economic growth to reduce overheating) and unemployment reduction (which stems from output growth). Both Table 5 and Table 6 show that fiscal policy is more relative more aggressive when economic growth is the policy priority, while monetary policy is relatively more aggressive when inflation is the priority.

5. Conclusion

This paper is the first paper to compute optimal fiscal and monetary policy in a wavelet-based model that utilizes a wavelet-decomposed version of OL. It is also the first paper to employ a discrete wavelet transformation to obtain an estimation of OL. Although Aguiar-Conraria, Martins, and Soares (2020) utilized a CWT to analyze the estimated form of OL, there has been no attempt to integrate this into a policy analysis, so once again this paper breaks new ground. Our paper goes a step further by providing insights into the effects on the labor market of optimal fiscal and monetary policy across different frequency ranges. Using the late-2018 U.S. economic conditions as reference point, we created a Matlab program to simulate optimal forecasts using a large-scale WBC model. The simulated results showed that fiscal policy was utilized relatively more aggressively to achieve a performance objective that increased the emphasis on economic growth. Monetary policy, on the other hand, was more aggressively utilized when the performance objective increased the emphasis on inflation.

The simulation results are not meant to serve as a fully integrated econometric forecast. Instead, the analysis is aimed shedding light on how OL is likely to affect optimal and monetary fiscal policy when the decomposed frequencies of the business cycle are considered. This is especially useful since it can augment central bank forecasting models and capture some of the political cycle considerations. One aspect that is not included in our model is any explicit modeling of labor productivity. As shown in the previous research on OL and monetary policy (Hudgins and Shuai, 2006; Aaronson et al., 2013; Eeckhout and Lindenlaub, 2019), this affects the response of unemployment of output asymmetrically across the business cycle. Thus, explicitly incorporating labor productivity within wavelet analyses of OL and in WBC models would be a further research avenue that would improve the link between the business cycle movements of the GDP components and unemployment.

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Appendix

Tables A1 and A2, respectively, show the estimation results for equations (7), (8), respectively. The natural rate of unemployment was taken to be 4.5%, and the potential GDP was estimated as a semi-logarithmic trend using data from the post-Bretton-Woods sample period.

Table A1

Okun's Law Unemployment coefficient estimates from equation (7), with *(p-values)*

$$un_{j,k} - un^*_{j,k} = \beta_{un,0} + \beta_{un,1}\ (\%\Delta Y_{j,k} - \%\Delta Y^*_{j,k}) + \omega_{un,j,k}$$

j	Quarters	$\beta_{un,j,0}$	$\beta_{un,j,1}$	R^2
1	2 to 4	1.7504 *(0.0000)*	-31.2070 *(0.0000)*	0.38
2	4 to 8	1.7466 *(0.0000)*	-34.5033 *(0.0000)*	0.39
3	8 to 16	1.7447 *(0.0000)*	-35.1798 *(0.0000)*	0.31
4	16 to 32	1.7593 *(0.0000)*	-23.2571 *(0.0000)*	0.15
5	32 to 64	1.7999 *(0.0000)*	-21.4013 *(0.0000)*	0.16

Table A2

Okun's Law Unemployment coefficient estimates from equation (8), with *(p-values)*

$$d_{un,j,k} = \beta_{un,j,0} + \beta_{un,j,1}\, d_{Y,j,k} + \omega_{un,j,k}$$

j	Quarters	$\beta_{un,j,0}$	$\beta_{un,j,1}$	R^2
1	2 to 4	0.000745 *(0.9122)*	-0.000841 *(0.0000)*	0.13
2	4 to 8	0.001510 *(0.8915)*	-0.001156 *0.000000*	0.45
3	8 to 16	0.006752 *(0.6689)*	-0.001264 *(0.0000)*	0.72
4	16 to 32	0.012397 *(0.5830)*	-0.001288 *(0.0000)*	0.82
5	32 to 64	0.047576 *(0.2128)*	-0.000952 *(0.0000)*	0.66

Since equation (7) is estimated as a modified version of OL where the output gap measured in percentage change form, rather than in logarithms, the results are not directly comparable to previous studies. After converting from percentages, the coefficients in table A1 are closer to the 0.3 value in Okun (1962) than the values of 0.5 in Ball, Leigh and Loungani (2017), and the values at lower frequency ranges (but only in the levels version of OL) of 0.8 in Aguiar-Conraria, Martins, and Soares (2020).

Our empirical results, especially for equation (8), do align with the general findings of Aguiar-Conraria, Martins, and Soares (2020), where the statistical significance and magnitude of the OL coefficients are larger for the medium cycle frequencies from 2 to 8 years have the largest impact on unemployment. Our reduced form state-space model contains a 1-quarter lag for most wavelet-decomposed variables, with other variables also including a 2-quarter lag. Thus, the manner in which the results are integrated within our WBC model is also consistent with Aguiar-Conraria, Martins, and Soares (2020), which finds that although the lead or lag relationship of output to unemployment does vary over time and across frequencies, a lag in the unemployment response of 4 months is generally sufficient. Moreover, our state-space model actually computes these output lags by recombining the GDP components in the first-order matrix difference equation system, which is consistent with Anderton et al. (2014), which finds that using GDP component data enhances the OL relationship.

Whereas the purpose of Aguiar-Conraria, Martins, and Soares (2020) is to improve the estimates of OL by utilizing wavelets to capture the time-frequency structure, the thrust of this analysis is show that a WBC model can be built which also captures the time-frequency aspects of OL. Thus, our model could employ the estimates obtained by any of these studies as the parameter values in the simulations. As a further extension, these coefficients could also be made time-varying within the model framework.