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Wavelet-based monetary and fiscal policy in the Euro area under optimal tracking control

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Supplemental Appendix (SA)

SA.1 Variable Definitions

Let the variable definition list be given as follows:

C_k	=	total real personal consumption expenditures in period *k*
C_k^*	=	desired consumption, or target consumption, in period *k*
I_k	=	real gross private domestic investment in period *k*
I_k^*	=	desired investment, or target investment, in period *k*
Y_k	=	real gross national product in period *k*
G_k	=	real government spending in period *k*
G_k^*	=	desired government spending in period *k*
NX_k	=	real net exports in period *k*

$d_{C,j,k}$ = the value of the consumption expenditure crystal for frequency j in quarter k, where $j = 1, \ldots, 5$

$d_{I,j,k}$ = the value of the private domestic investment crystal for frequency j in quarter k, where $j = 1, \ldots, 5$

$d_{G,j,k}$ = the value of the government spending crystal for frequency j in quarter k, where $j = 1, \ldots, 5$

$S_{C5,k}$ = the value of the consumption modified smooth in quarter k, where $J = 5$

$S^*_{C5,k}$ = target value for the modified smooth trend consumption, in period k

$S_{I5,k}$ = the value of the investment modified smooth in quarter k, where $J = 5$

$S^*_{I5,k}$ = target value for the modified smooth trend investment, in period k

$S_{G5,k}$ = the value of the government spending modified smooth in quarter k, where $J = 5$

$S^*_{G5,k}$ = target value for the modified smooth trend in government spending, in period k

$C_{j,k}$ = the prevailing consumption expenditure at frequency j in quarter k, which includes the sum of the consumption crystal and the consumption modified smooth, where $j = 1, \ldots, 5$

$C^*_{j,k}$ = the target consumption expenditure at frequency range j in quarter k

$I_{j,k}$ = the prevailing private domestic investment at frequency j in quarter k, which includes the sum of the investment crystal and the investment modified smooth, where $j = 1, \ldots, 5$

$I^*_{j,k}$ = the target investment expenditure at frequency range j in quarter k

$G_{j,k}$ = the prevailing government spending at frequency range j in quarter k, which includes the sum of the government spending crystal and the government purchases spending modified smooth, where $j = 1, \ldots, 5$

$G^*_{j,k}$ = the target government expenditure at frequency range j in quarter k

$G^d_{j,k}$ = the current cycle trend government spending at frequency range j in quarter k.

T_k = net government taxes and income in quarter k, which equals total government tax and income minus total government transfer payments.

DEF_k = total government budget deficit in quarter k, which equals government spending minus net government taxes

$DEBT_k$ = total government debt in quarter k

ig_k = average quarterly interest rate on government debt in quarter k

ir_k = 3-month (quarterly) nominal euro market interest rate in quarter k

$ir_{j,k}$ = the prevailing 3-month (quarterly) euro market interest rate at frequency j in quarter k, which includes the sum of the interest rate crystals and the interest rate modified smooth, where $j = 1, \ldots, 5$

$I^*_{j,k}$ = the target investment expenditure at frequency range j in quarter k

τ_k = rate of net tax (tax minus transfers) collection in quarter k

SA.2 Model Estimation

The model procedure assumes that the economy would have some reaction to any announced, consistent government policy regime. Since no such control policy has yet been historically implemented, there was no past distinction between government spending under the optimal control policy, and the spending trajectory along the current cycle trajectory that reflects existing expectations. The rational reactions involving an adjustment for government debt under the lack of any announced consistent policy are zero, since there was no such policy against which to react. Thus, the lag of the current government spending trend variable and the lag of government debt variables are not included in equations (A11) and (A12). Instead, the values for these coefficients are assigned and evaluated under different scenarios in control system policy simulations.

Tables SA1 – SA3 show that the estimated equations for all of the consumption, investment, and government spending equations over each frequency range have a good fit. All of the coefficients have the expected sign, and almost all of the coefficients are statistically significant. The consumption equation coefficients in table SA1 show that both investment and government spending have a crowding-in effect on consumption. The investment coefficients in table SA2 show that consumption has a crowding-in effect, but government spending has a crowing-out effect on investment. The government spending coefficients in table 4 shows that the average quarterly growth rate is close to .005 per quarter (about 2% per year) at all frequency ranges.

Table SA1

Estimated coefficients for $C_{j,k}$ at each frequency ($\boldsymbol{R}^2 > 0.99$ for all equations, $j = 1, \ldots, 5$)

j	Constant $_j$	Coefficient $C_{j,k-1}$	Coefficient $I_{j,k-1}$	Coefficient $G_{j,k-1}$	Coefficient $ir_{j,k-1}$
1	42438.01	0.9322	0.1056	-0.0063	-1628.9254
t-statistic	*3.445*	*14.306*	*2.145*	*-0.057*	-3.903
2	45061.74	0.9140	0.1229	0.0199	-1730.7089
t-statistic	*4.485*	*16.841*	*2.945*	*0.221*	-5.126
3	48275.35	0.8815	0.1524	0.0704	-1744.7664
t-statistic	*5.919*	*19.893*	*4.365*	*0.963*	-6.472
4	30363.70	0.9943	0.0535	-0.0981	-975.7151
t-statistic	*3.374*	*21.349*	*1.427*	*-1.311*	-3.295
5	22607.80	1.0942	-0.0524	-0.2459	-459.4918
t-statistic	*2.131*	*23.808*	*-1.345*	*-3.616*	-1.210

Table SA2

Estimated coefficients for $I_{j,k}$ at each frequency ($\boldsymbol{R}^2 > 0.96$ for all equations, $j = 1, \ldots, 5$)

j	Constant $_j$	Coefficient $I_{j,k-1}$	Coefficient $G_{j,k-1}$	Coefficient $ir_{j,k-1}$
1	38221.18	1.0296	-0.1074	-1708.0304
t-statistic	*2.618*	*23.563*	*-1.863*	*-2.465*
2	41403.18	1.0556	-0.1401	-2028.6926
t-statistic	*3.718*	*31.213*	*-3.206*	*-3.851*
3	36408.31	1.0384	-0.1128	-1656.9915
t-statistic	*3.629*	*33.433*	*-3.005*	*-3.610*
4	30481.04	1.0071	-0.0708	-989.9166
t-statistic	*2.396*	*28.166*	*-1.701*	*-1.748*
5	37241.97	1.0035	-0.0836	-1011.3181
t-statistic	*2.371*	*29.703*	*-1.713*	*-1.282*

Table SA3

Estimated coefficients for $G_{j,k}$ at each frequency

j	*Coefficient* $G_{j,k-1}$	R^2
1	1.0037	
t-statistic	*3295.88*	0.9999
2	1.0037	
t-statistic	*4507.06*	0.9999
3	1.0037	
t-statistic	*4519.06*	0.9999
4	1.0037	
t-statistic	*5082.43*	0.9999
5	1.0036	
t-statistic	*4283.56*	0.9999

Table SA4 gives the paths for the modified smooth trends after extracting the crystals from all 5 frequency ranges for consumption, investment, government spending, and the 3-month interest rate, as specified in equations (A19), (A20), (A21), and (A22), respectively. The summation of the two coefficients in each of the equations forms a weighted average trend growth rate. In consumption trend series equation, the coefficient on the lagged value of the series is $s_{C,1} = 0.89$, which is much larger than coefficient on the lagged value of aggregate consumption, given by $s_{C,2} = 0.11$. This pattern holds for the investment and government spending modified smooth trend series, where the coefficients on the lagged value of both series is over 0.8, while the coefficients on the lagged aggregate investment and aggregate government spending are less than 0.2. All three equations obtain a good fit, with statistically significant coefficients.

Table SA4

Estimated coefficients for the *modified smooth* trend residuals for *Consumption*, *Investment*, and *Government Purchases* at each frequency range

	$s_{C,1}$	$s_{C,2}$	R^2
$S_{C5,k}$	0.8927	0.1133	
t-statistic	*48.8170*	*6.2208*	0.9999

	$s_{I,1}$	$s_{I,2}$	R^2
$S_{I5,k}$	0.8194	0.1861	
t-statistic	*37.5588*	*8.5257*	0.9967

	$s_{G,1}$	$s_{G,2}$	R^2
S_{G5}	0.8609	0.1441	
t-statistic	*46.0681*	*7.7119*	0.9997

	$s_{ir,1}$	$s_{ir,2}$	R^2
S_{ir5}	0.7966	0.2202	
t-statistic	*22.1258*	*0.0415*	0.9812

SA.3 State-Space Specification

We transform the LQ-tracking problem into a LQ-regulator problem using the procedure in Crowley and Hudgins (2015), thus creating a state-space with 80 state variables, 80 state equations, and 10 control variables. Although this transformation creates a higher dimensional state-space, it greatly simplifies the subsequent solution procedures for deterministic, stochastic, and H^{∞}- optimal control problems. This conversion method is similar to that used in Hudgins and Na (2016). Although this is a large scale system, the state-space construction procedures and the accompanying MATLAB program that we have developed have proven to be efficient and feasible to employ. This wavelet-based system framework can easily be adopted within the context of larger base models, although the inclusion of the different frequency ranges would substantially increase the size of the larger econometric models.

The present transformation uses the Crowley and Hudgins (2015) and Hudgins and Na (2016) approach of embedding the constant terms within the state equations. The 80-dimensional state vector is defined as follows:

$$x_k = \left[x_{1,k}\ ;\ x_{2,k}\ ;\ \ldots\ ;\ x_{80,k}\right]^T \qquad \text{(SA1)}$$

where

$$x_k = [\ C_{1,k};\ C_{2,k};C_{3,k};C_{4,k};C_{5,k};S_{C5,k};\ I_{1,k};\ I_{2,k};I_{3,k};I_{4,k};I_{5,k};S_{I5,k};\ c_k;$$
$$C^*_{1,k};C^*_{2,k};C^*_{3,k};C^*_{4,k};C^*_{5,k};I^*_{1,k};I^*_{2,k};I^*_{3,k};I^*_{4,k};I^*_{5,k};G^*_{1,k};G^*_{2,k};G^*_{3,k};G^*_{4,k};G^*_{5,k};$$
$$\hat{G}^d_{1,k};\hat{G}^d_{2,k};\hat{G}^d_{3,k};\hat{G}^d_{4,k};\hat{G}^d_{5,k};S_{G5,k};C_k;I_k;G_k;C^*_k;I^*_k;G^*_k;NX_k;Y_k;$$
$$T_k;DEF_k;DEBT_k;G_{1,k-1};G_{2,k-1};G_{3,k-1};G_{4,k-1};G_{5,k-1};$$
$$G_{1,k-1}-G_{1,k-2};G_{2,k-1}-G_{2,k-2};G_{3,k-1}-G_{3,k-2};G_{4,k-1}-G_{4,k-2};G_{5,k-1}-G_{5,k-2};$$
$$(G_{1,k-1}-G_{1,k-2})^*;(G_{2,k-1}-G_{2,k-2})^*;(G_{3,k-1}-G_{3,k-2})^*;(G_{4,k-1}-G_{4,k-2})^*;$$
$$(G_{5,k-1}-G_{5,k-2})^*;\ DEF^*_k;\ DEBT^*_k\ ;S^*_{C5,k};S^*_{I5,k};S^*_{G5,k}\ ;$$
$$S^*_{ir5,k};S_{ir5,k};\ ir^*_k;\ ir_{k-1};\ ir_{1,k-1};\ ir_{2,k-1};\ ir_{3,k-1};\ ir_{4,k-1};\ ir_{5,k-1};$$
$$ir_{1,k-1}-ir_{1,k-2};ir_{2,k-1}-ir_{2,k-2};ir_{3,k-1}-ir_{3,k-2};ir_{4,k-1}-ir_{4,k-2};ir_{5,k-1}-ir_{5,k-2};$$
$$ir_{k-2}]^T$$

Define the control vector so that the first five elements are difference between the actual and targeted level of government spending and the last five elements are the tracking errors for the short-term market interest rate at each frequency range:

$$u_k = \left[u_{G1,k};u_{G2,k};u_{G3,k};u_{G4,k};u_{G5,k}|u_{ir1,k};u_{ir2,k};u_{ir3,k};u_{ir4,k};u_{ir5,k}\right]^T \quad \text{(SA2)}$$
$$u_{G\,j,k} = G_{j,k} - G^*_{j,k} \qquad u_{ir\,j,k} = ir_{j,k} - ir^*_k$$

The disturbance vector for stochastic and robust design cases is defined by (SA3), where the vector is 0 for the deterministic case.

$$\omega_k = \left[\omega_{1,1,k};\ \omega_{1,2,k};\ \omega_{1,3,k};\omega_{1,4,k};\omega_{1,5,k}\,|\,\omega_{2,1,k};\ \omega_{2,2,k};\ \omega_{2,3,k}; \right. \quad \text{(SA3)}$$
$$\left. \omega_{2,4,k};\omega_{2,5,k}\,|\,\omega_{3,1,k};\ \omega_{3,2,k};\ \omega_{3,3,k};\omega_{3,4,k};\omega_{3,5,k};\omega_{4,k};\omega_{5,k};\omega_{6,k};\omega_{7,k}\right]^T$$

Since each of the first five control variables in the vector u_k include the negative of the targeted levels of government spending, and each of the last 5 variables include the negative of the targeted interest rate at each frequency, these target variables are added to the 5 state equations for the individual frequencies of consumption and the 5 state equations for the individual frequencies for investment. The net effect of adding and subtracting the same variable is 0, but this allows the problem to be written in standard LQ-regulator format. Once the optimal control has been simulated to produce the values for $u_{G\,j,k}$ and $u_{ir\,j,k}$ over each frequency range, the target level of government spending and the interest rate, $G^*_{j,k}$ and $ir^*_{j,k}$, will have to be added to $u_{G\,j,k}$ and $u_{ir\,j,k}$, respectively, in order to recover the values for government spending, $G_{j,k}$, and the interest rate, $ir_{j,k}$, over each frequency range. However, these values are also automatically recovered with one lag in state equations 46 – 50 and 70 – 74, respectively, by adding the target values to the state values of government spending and the interest rate.

The matrix state-space equation system is now given by (SA4).

$$x_{k+1} = A_k x_k + B_k u_k + D_k \omega_k \tag{SA4}$$

dim x = (80, 1) *dim u* = (10, 1) *dim* ω = (19, 1)
dim A = (80, 80) *dim B* = (80, 10) *dim D* = (80, 19)

The 80 state equations are listed as follows:

$$x_{1,k+1} = \delta_{1,1} x_{1,k} + \delta_{2,1} x_{7,k} + \delta_{0,1} x_{13,k} + \delta_{3,1} x_{24,k} + \delta_{5,1} x_{29,k} + \delta_{6,1} x_{45,k} + \delta_{4,1} x_{68,k} + \delta_{3,1} u_{G,1,k} + \delta_{4,1} u_{ir,1,k} + \delta_{7,1} \omega_{1,1,k}$$

$$x_{2,k+1} = \delta_{1,2} x_{2,k} + \delta_{2,2} x_{8,k} + \delta_{0,2} x_{13,k} + \delta_{3,2} x_{25,k} + \delta_{5,2} x_{30,k} + \delta_{6,2} x_{45,k} + \delta_{4,2} x_{68,k} + \delta_{3,2} u_{G,2,k} + \delta_{4,2} u_{ir,2,k} + \delta_{7,2} \omega_{1,2,k}$$

$$x_{3,k+1} = \delta_{1,3} x_{3,k} + \delta_{2,3} x_{9,k} + \delta_{0,3} x_{13,k} + \delta_{3,3} x_{26,k} + \delta_{5,3} x_{31,k} + \delta_{6,3} x_{45,k} + \delta_{4,3} x_{68,k} + \delta_{3,3} u_{G,3,k} + \delta_{4,3} u_{ir,3,k} + \delta_{7,3} \omega_{1,3,k}$$

$$x_{4,k+1} = \delta_{1,4} x_{4,k} + \delta_{2,4} x_{10,k} + \delta_{0,4} x_{13,k} + \delta_{3,4} x_{27,k} + \delta_{5,4} x_{32,k} + \delta_{6,4} x_{45,k} + \delta_{4,4} x_{68,k} + \delta_{3,4} u_{G,4,k} + \delta_{4,4} u_{ir,4,k} + \delta_{7,4} \omega_{1,4,k}$$

$$x_{5,k+1} = \delta_{1,5} x_{5,k} + \delta_{2,5} x_{11,k} + \delta_{0,5} x_{13,k} + \delta_{3,5} x_{28,k} + \delta_{5,5} x_{33,k} + \delta_{6,5} x_{45,k} + \delta_{4,5} x_{68,k} + \delta_{3,5} u_{G,5,k} + \delta_{4,5} u_{ir,5,k} + \delta_{7,5} \omega_{1,5,k}$$

$$x_{6,k+1} = s_{C,1} x_{6,k} + s_{C,2} x_{35,k} + s_{C,3} \omega_{4,k}$$

$$x_{7,k+1} = \lambda_{1,1} x_{7,k} + \lambda_{0,1} x_{13,k} + \lambda_{2,1} x_{24,k} + \lambda_{3,1} x_{68,k} + \lambda_{3,1} u_{ir,1,k} + \lambda_{4,1} \omega_{2,1,k}$$

$$x_{8,k+1} = \lambda_{1,2} x_{8,k} + \lambda_{0,2} x_{13,k} + \lambda_{2,2} x_{25,k} + \lambda_{3,2} x_{68,k} + \lambda_{3,2} u_{ir,2,k} + \lambda_{4,2} \omega_{2,2,k}$$

$$x_{9,k+1} = \lambda_{1,3} x_{9,k} + \lambda_{0,3} x_{13,k} + \lambda_{2,3} x_{26,k} + \lambda_{3,3} x_{68,k} + \lambda_{3,3} u_{ir,3,k} + \lambda_{4,3} \omega_{2,3,k}$$

$$x_{10,k+1} = \lambda_{1,4} x_{10,k} + \lambda_{0,4} x_{13,k} + \lambda_{2,4} x_{27,k} + \lambda_{3,4} x_{68,k} + \lambda_{3,4} u_{ir,4,k} + \lambda_{4,4} \omega_{2,4,k}$$

$$x_{11,k+1} = \lambda_{1,5} x_{11,k} + \lambda_{0,5} x_{13,k} + \lambda_{2,5} x_{28,k} + \lambda_{3,5} x_{68,k} + \lambda_{3,5} u_{ir,5,k} + \lambda_{4,5} \omega_{2,5,k}$$

$$x_{12,k+1} = s_{I,1} x_{12,k} + s_{I,2} x_{36,k} + s_{I,3} \omega_{5,k}$$

$$x_{13,k+1} = x_{13,k}$$

$$x_{14,k+1} = (1+ g_{C,1,k}) x_{14,k}$$

$$x_{15,k+1} = (1+ g_{C,2,k}) x_{15,k}$$

$$x_{16,k+1} = (1+ g_{C,3,k}) x_{16,k}$$

$$x_{17,k+1} = (1+ g_{C,4,k}) x_{17,k}$$

$$x_{18,k+1} = (1+g_{C,5,k})\,x_{18,k}$$
$$x_{19,k+1} = (1+g_{I,1,k})\,x_{19,k}$$
$$x_{20,k+1} = (1+g_{I,2,k})\,x_{20,k}$$
$$x_{21,k+1} = (1+g_{I,3,k})\,x_{21,k}$$
$$x_{22,k+1} = (1+g_{I,4,k})\,x_{22,k}$$
$$x_{23,k+1} = (1+g_{I,5,k})\,x_{23,k}$$
$$x_{24,k+1} = (1+g_{G,1,k})\,x_{24,k}$$
$$x_{25,k+1} = (1+g_{G,2,k})\,x_{25,k}$$
$$x_{26,k+1} = (1+g_{G,3,k})\,x_{26,k}$$
$$x_{27,k+1} = (1+g_{G,4,k})\,x_{27,k}$$
$$x_{28,k+1} = (1+g_{G,5,k})\,x_{28,k}$$
$$x_{29,k+1} = \rho_{1,1}x_{24,k} + \rho_{1,1}u_{G,1,k} + \rho_{2,1}\omega_{3,1,k-1}$$
$$x_{30,k+1} = \rho_{1,2}x_{25,k} + \rho_{1,2}u_{G,2,k} + \rho_{2,2}\omega_{3,2,k-1}$$
$$x_{31,k+1} = \rho_{1,3}x_{26,k} + \rho_{1,3}u_{G,3,k} + \rho_{2,3}\omega_{3,3,k-1}$$
$$x_{32,k+1} = \rho_{1,4}x_{27,k} + \rho_{1,4}u_{G,4,k} + \rho_{2,4}\omega_{3,4,k-1}$$
$$x_{33,k+1} = \rho_{1,5}x_{28,k} + \rho_{1,5}u_{G,5,k} + \rho_{2,5}\omega_{3,5,k-1}$$
$$x_{34,k+1} = s_{G,1}x_{34,k} + s_{G,2}x_{37,k} + s_{G,3}\omega_{6,k}$$
$$\begin{aligned} x_{35,k+1} = {} & \delta_{1,1}x_{1,k} + \delta_{1,2}x_{2,k} + \delta_{1,3}x_{3,k} + \delta_{1,4}x_{4,k} + \delta_{1,5}x_{5,k} - 4x_{6,k} + \delta_{2,1}x_{7,k} \\ & + \delta_{2,2}x_{8,k} + \delta_{2,3}x_{9,k} + \delta_{2,4}x_{10,k} + \delta_{2,5}x_{11,k} + \sum_{j=1}^{5}\delta_{0,j}x_{13,k} + \delta_{3,1}x_{24,k} \\ & + \delta_{3,2}x_{25,k} + \delta_{3,3}x_{26,k} + \delta_{3,4}x_{27,k} + \delta_{3,5}x_{28,k} + \delta_{5,1}x_{29,k} + \delta_{5,2}x_{30,k} \\ & + \delta_{5,3}x_{31,k} + \delta_{5,4}x_{32,k} + \delta_{5,5}x_{33,k} + \sum_{j=1}^{5}\delta_{6,j}x_{45,k} + \sum_{j=1}^{5}\delta_{4,j}x_{68,k} \\ & + \sum_{j=1}^{5}\delta_{3,j}u_{G,j,k} + \sum_{j=1}^{5}\delta_{4,j}u_{ir,j,k} + \sum_{j=1}^{5}\delta_{7,j}\omega_{1,j,k} \end{aligned}$$

$$\begin{aligned} x_{36,k+1} = {} & \lambda_{1,1}x_{7,k} + \lambda_{1,2}x_{8,k} + \lambda_{1,3}x_{9,k} + \lambda_{1,4}x_{10,k} + \lambda_{1,5}x_{11,k} - 4x_{12,k} \\ & + \sum_{j=1}^{5}\lambda_{0,j}x_{13,k} + \lambda_{2,1}x_{24,k} + \lambda_{2,2}x_{25,k} + \lambda_{2,3}x_{26,k} + \lambda_{2,4}x_{27,k} + \lambda_{2,5}x_{28,k} \\ & + \sum_{j=1}^{5}\lambda_{3,j}x_{68,k} + \sum_{j=1}^{5}\lambda_{2,j}u_{G,j,k} + \sum_{j=1}^{5}\lambda_{3,j}u_{ir,j,k} + \sum_{j=1}^{5}\lambda_{4,j}\omega_{2,j,k} \end{aligned}$$

$$x_{37,k+1} = x_{24,k}+x_{25,k}+x_{26,k}+x_{27,k}+x_{28,k}-4x_{34,k}+\sum_{j=1}^{5}u_{G,j,k}$$

$$x_{38,k+1} = (1+g_{C,k})\,x_{38,k}$$

$$x_{39,k+1} = (1+g_{I,k})\,x_{39,k}$$

$$x_{40,k+1} = (1+g_{G,k})\,x_{40,k}$$

$$x_{41,k+1} = n_0\,x_{13,k}$$

$$x_{42,k+1} = x_{35,k} + x_{36,k} + x_{37,k} + x_{41,k}$$

$$x_{43,k+1} = \tau\,x_{42,k}$$

$$x_{44,k+1} = x_{24,k}+x_{25,k}+x_{26,k}+x_{27,k}+x_{28,k}-4x_{34,k}-x_{43,k} + \sum_{j=1}^{5}u_{G,j,k}$$

$$x_{45,k+1} = x_{44,k} + (1+i)\,x_{45,k}$$

$$x_{46,k+1} = x_{24,k} + u_{G,1,k}$$

$$x_{47,k+1} = x_{25,k} + u_{G,2,k}$$

$$x_{48,k+1} = x_{26,k} + u_{G,3,k}$$

$$x_{49,k+1} = x_{27,k} + u_{G,4,k}$$

$$x_{50,k+1} = x_{28,k} + u_{G,5,k}$$

$$x_{51,k+1} = x_{24,k} - x_{46,k} + u_{G,1,k}$$

$$x_{52,k+1} = x_{25,k} - x_{47,k} + u_{G,2,k}$$

$$x_{53,k+1} = x_{26,k} - x_{48,k} + u_{G,3,k}$$

$$x_{54,k+1} = x_{27,k} - x_{49,k} + u_{G,4,k}$$

$$x_{55,k+1} = x_{28,k} - x_{50,k} + u_{G,5,k}$$

$$x_{56,k+1} = g_{G,1,k}\,x_{24,k}$$

$$x_{57,k+1} = g_{G,2,k}\,x_{25,k}$$

$$x_{58,k+1} = g_{G,3,k}\,x_{26,k}$$

$$x_{59,k+1} = g_{G,4,k}\,x_{27,k}$$

$$x_{60,k+1} = g_{G,5,k}\,x_{28,k}$$

$$x_{61,k+1} = (1+g_{DEF,k})\,x_{61,k}$$

$$x_{62,k+1} = (1+g_{DEBT,k})\,x_{62,k}$$

$$x_{63,k+1} = (1+g_{S,C5,k})\,x_{63,k}$$

$$x_{64,k+1} = (1+g_{S,I5,k})\,x_{64,k}$$
$$x_{65,k+1} = (1+g_{S,G5,k})\,x_{65,k}$$
$$x_{66,k+1} = (1+g_{S,ir5,k})\,x_{66,k}$$
$$x_{67,k+1} = s_{ir,1}x_{67,k} + s_{ir,2}x_{69,k} + s_{ir,3}\,\omega_{7,k}$$
$$x_{68,k+1} = (1+g_{ir,k})\,x_{68,k}$$
$$x_{69,k+1} = 5x_{68,k} - 4x_{67,k} + \sum_{j=1}^{5} u_{ir,j,k}$$
$$x_{70,k+1} = x_{68,k} + u_{ir,1,k}$$
$$x_{71,k+1} = x_{68,k} + u_{ir,2,k}$$
$$x_{72,k+1} = x_{68,k} + u_{ir,3,k}$$
$$x_{73,k+1} = x_{68,k} + u_{ir,4,k}$$
$$x_{74,k+1} = x_{68,k} + u_{ir,5,k}$$
$$x_{75,k+1} = x_{68,k} - x_{70,k} + u_{ir,1,k}$$
$$x_{76,k+1} = x_{68,k} - x_{71,k} + u_{ir,2,k}$$
$$x_{77,k+1} = x_{68,k} - x_{72,k} + u_{ir,3,k}$$
$$x_{78,k+1} = x_{68,k} - x_{73,k} + u_{ir,4,k}$$
$$x_{79,k+1} = x_{68,k} - x_{74,k} + u_{ir,5,k}$$
$$x_{80,k+1} = x_{69,k}$$

SA.4 Transformed Deterministic Regulator Design

Consider the deterministic LQ regulator problem where the disturbance vector is zero, or $\omega_k = 0$, or alternatively, where the disturbance coefficient vector is $D_k = 0$. After rewriting expression (4) based on the state space system in (SA4), the objective is to minimize the performance index

$$\min_{u} J(u) = x_{K+1}^T Q_f\, x_{K+1} + \sum_{k=1}^{K}\left[x_k^T Q_k x_k + u_k^T R_k u_k \right] \tag{SA5}$$

subject to

$$x_{k+1} = A_k x_k + B_k u_k \quad ; \quad x(1) = x_1 \tag{SA6}$$

where the size of the penalty weighting matrices are

$dim\, Q_f = (80, 80) \quad dim\, Q_k = (80, 80) \quad dim\, R_k = (10, 10)$

The solution to the LQ regulator problem is found by first computing the recursive equations (SA7) and (SA8) offline in retrograde time.

$$F_k = \left(B_k^T P_{k+1} B_k + R_k \right)^{-1} B_k^T P_{k+1} A_k \tag{SA7}$$

$$P_k = Q_k + A_k^T P_{k+1} \left(A_k - B_k F_k \right); \qquad P_{k+1} = Q_f \tag{SA8}$$

These recursive equations are much simpler to compute than the longer recursive equations employed by Chow (1975), Kendrick (1981), and others that arise when solving the LQ-tracking problem. Using the values computed in (SA7) and (SA8), the unique optimal feedback control policy is computed in forward time by

$$u_k^{Optimal} = -F_k x_k \tag{SA9}$$

The optimal closed-loop state trajectory is given by

$$x_{k+1} = \left(A_k - B_k F_k \right) x_k \; ; \qquad x(1) = x_1 \tag{SA10}$$

The control equations in (SA7) and (SA8) are the same for the stochastic LQG form of the model with perfect state information. The state variable trajectory, however, would be calculated by equation (SA4), rather than (SA10). The control vector in equation (SA9) would then be computed by using equations (SA7) and (SA4).

SA.5 Model Simulation Parameters

The penalty parameter coefficients for the performance index tracking errors in equation (4) are given in Table SA5 under each of the three policy emphasis scenarios. The simulations define the initial values for the state variables in period 1 to correspond to the Euro area quarterly data in 2014, quarter 3, measured in billions of euros. Net exports are set at a constant value of n_0 = 108.57. The stock of government debt is set at $DEBT_0$ = 8,192.9902, which is 92.1% of the initial real GDP value of Y_0= 2,220.4352. Since the Euro area member states had an average budget deficit of 3% of output, the initial budget deficit is set at DEF_0 = 66.61306. Given the initial government spending value of G_0= 466.745, the government spending minus the deficit yields an initial value for net taxes of T_0 = 400.13204. This amount is 18% of total output, so the net tax rate is set at τ_0 = 0.18. The quarterly interest rate on the debt is set at ig_0 = .005, which is 2% per year. In equation (10), the weight for the current level of government purchases in the expectation formation equation is set at $\phi_{j,k}$ = 0.90 and the parameter weight for the adjustment for the national debt differential in the expectation is set at $\pi_{j,k}$ = 0.0005 for all frequency ranges in all periods.

Table SA5

Performance Index Coefficients

	Emphasis		
	Dual	Fiscal	Monetary
$q_{1,f}$ =	2.0	2.0	2.0
$q_{2,f}$ =	2.0	2.0	2.0
$q_{1,k}$ =	0.2	0.2	0.2
$q_{2,k}$ =	0.2	0.2	0.2
$q_{3,1,f}$ =	1.0	1.0	1.0
$q_{3,2,f}$ =	4.0	4.0	4.0
$q_{3,3,f}$ =	16.0	16.0	16.0
$q_{3,4,f}$ =	16.0	16.0	16.0
$q_{3,5,f}$ =	1.0	1.0	1.0
$q_{3,1,k}$ =	0.1	0.1	0.1
$q_{3,2,k}$ =	0.4	0.4	0.4
$q_{3,3,k}$ =	1.6	1.6	1.6
$q_{3,4,k}$ =	1.6	1.6	1.6
$q_{3,5,k}$ =	0.1	0.1	0.1
$q_{4,1,f}$ =	1.0	1.0	1.0
$q_{4,2,f}$ =	4.0	4.0	4.0
$q_{4,3,f}$ =	16.0	16.0	16.0
$q_{4,4,f}$ =	16.0	16.0	16.0
$q_{4,5,f}$ =	1.0	1.0	1.0
$q_{4,1,k}$ =	0.1	0.1	0.1
$q_{4,2,k}$ =	0.4	0.4	0.4
$q_{4,3,k}$ =	1.6	1.6	1.6
$q_{4,4,k}$ =	1.6	1.6	1.6
$q_{4,5,k}$ =	0.1	0.1	0.1
$q_{5,k}$ =	0.2	0.2	0.2
$q_{6,k}$ =	0.2	0.2	0.2

	Emphasis		
	Dual	Fiscal	Monetary
$q_{7,1,k}$ =	0.2	0.2	0.2
$q_{7,2,k}$ =	0.2	0.2	0.2
$q_{7,3,k}$ =	0.2	0.2	0.2
$q_{7,4,k}$ =	0.2	0.2	0.2
$q_{7,5,k}$ =	0.2	0.2	0.2
$q_{8,k}$ =	20.0	20.0	160.0
$q_{9,k}$ =	1,000,000,000,000	100,000,000,000,000	1,000,000,000,000
$q_{10,1,k}$ =	10,000,000,000,000	10,000,000,000,000	10,000,000,000,000
$q_{10,2,k}$ =	10,000,000,000,000	10,000,000,000,000	10,000,000,000,000
$q_{10,3,k}$ =	10,000,000,000,000	10,000,000,000,000	10,000,000,000,000
$q_{10,4,k}$ =	10,000,000,000,000	10,000,000,000,000	10,000,000,000,000
$q_{10,5,k}$ =	10,000,000,000,000	10,000,000,000,000	10,000,000,000,000
$q_{11,k}$ =	100,000,000,000	100,000,000,000,000	100,000,000,000
$q_{S,C5,f}$ =	2.0	2.0	2.0
$q_{S,I5,f}$ =	2.0	2.0	2.0
$q_{S,C5,k}$ =	0.2	0.2	0.2
$q_{S,I5,k}$ =	0.2	0.2	0.2
$q_{S,G5,k}$ =	0.2	0.2	0.2
$q_{S,ir5,k}$ =	100,000,000	100,000,000	100,000,000
$r_{G,1,k}$ =	10.0	10.0	10.0
$r_{G,2,k}$ =	10.0	10.0	10.0
$r_{G,3,k}$ =	20.0	20.0	20.0
$r_{G,4,k}$ =	20.0	20.0	20.0
$r_{G,5,k}$ =	10.0	10.0	10.0
$r_{ir,1,k}$ =	100,000	100,000	100,000
$r_{ir,2,k}$ =	100,000	100,000	100,000
$r_{ir,3,k}$ =	200,000	200,000	200,000
$r_{ir,4,k}$ =	200,000	200,000	200,000
$r_{ir,5,k}$ =	200,000	200,000	200,000